Problem-1 (40%)
Consider a steam turbine power plant as shown with the given properties below. Enter state of pump is compressed liquid and isentropic pump efficiency is 75%. Assume no heat exchange between turbine and ambient and between pump and ambient (adiabatic process). And also neglecting any changes in kinetic and potential energies.

**calculate:**

a. The maximum turbine work output rate for mass flow rate of 100 kg/s.

b. The turbine isentropic efficiency if the turbine exit state is saturated vapor.

c. The pump work input rates and enthalpy values (isentropic and adiabatic) at the pump exit states

d. The thermal efficiency of the cycle

e. Draw the cycle in T-s diagrams and label states properly as isentropic and adiabatic states.

**Properties:** $P_1 = P_4 = 20$ MPa, $T_1 = 800$ °C, $P_2 = P_3 = 30$ kPa

![Diagram](image.png)

**Solution:**

Process: 1-2 and 3-4 (isentropic, $Q=0, \Delta S = 0$), 1-2' and 3-4'(adiabatic, $Q=0, \Delta S \neq 0$)

$h_1=4069.80 \text{ kJ/kg}$, $s_1 = s_2 = 7.0544 \text{ kJ/kg-K}$ (30 KPa $s_g=7.7686 \text{ kJ/kg-K}$), So, state-2 is saturated (mix state).

Find quality: $s_2=s_f + x s_g$,

$x = (7.0544-0.9439/6.8247)=89.5\%$

$h_2 = h_f + x h_f = 289.21 + 0.895*2336.07 = 2379.99 \text{ kJ/kg}$

a. System: Turbine: Continuity eq: $\dot{m}_2 = \dot{m}_1 = \dot{m} = 100 \text{ kg/s}$

First law (open system): $w_{T(max)} = h_1 - h_2 = 4069.80 - 2379.99 = 1689.81 \text{ kJ/kg}$

The max. turbine work output rate, $W = \dot{m} w_{T(max)} = 168981 \text{ KW} \approx 169 \text{ MW}$

b. $\eta_T = \frac{w_{T(adiabatic)}}{w_{T(isentropic)}} = \frac{h_1 - h_2}{1689.81} \quad h_2 = h_g = 2625.28 \text{ kJ/kg}$

$\eta_T = 85.5\%$
c. System: Pump  
Continuity eq: \( m_2 = m_1 = \dot{m} = 100 \text{ kg/s} \)

First law (open system):  
\[-w_p(\text{isentropic}) = h_4 - h_3 = v(P_4 - P_3) \quad v = v_f = 0.001022 \text{ m}^3/\text{kg}, \quad h_3 = h_4 = 289.21 \text{ kJ/kg} \]
\[-w_p = 0.001022(20000 - 30) = 20.41 \text{ kJ/kg} = h_4 - 289.21 \]
\[ h_4 = 309.62 \text{ kJ/kg} \]

\[ \eta_p = \frac{w_p(\text{isentropic})}{w_p(\text{adiabatic})} = \frac{h_4 - h_3}{h_4 - h_3} = \frac{20.41}{289.1 - 289.1} = 0.75, \quad h_4 = 316.31 \text{ kJ/kg} \]

\[ d. \quad \eta_{TH} = \frac{\sum w}{q_H} = \frac{w_f - w_p}{h_1 - h_4} = \frac{1689.81 \times 0.855 - 20.41/0.75}{4069.80 - 316.31} = 37.8\% \]

Problem-2 (20%)  
A house in the winter would be heated with a heat pump. The house is to be maintained at 21°C at all times. When the ambient temperature outside at the time of the coldest night drops to -16°C, the minimum electrical power required to drive the heat pump is 5 KW. What would be the maximum rate of heat lost from the house?

Solution:

\[ \beta' = \frac{\dot{Q}_H}{\dot{W}_L} = \frac{T_H}{T_H - T_L} \]
\[ \beta' = \frac{273 + 21}{21 - (-16)} = 7.95 \]
\[ \dot{Q}_H = 7.95 \times 5 = 39.75 \text{ KW} = \dot{Q}_{\text{leak}} \]
Problem-3(20%)
A large power plant 5 MW of 90°C waste-heat is the low temperature $Q_L$ input to a heat pump that upgrades it to a $Q_H$ delivered at 120°C. The process is said to occur with a power input of 0.4 MW. What does the second law say about this?

Solution:

C.V. The heat pump.

Energy Eq.: $\dot{Q}_H = \dot{Q}_L + \dot{W}_{HP}$

Definition of COP: $\beta_{HP} = \frac{\dot{Q}_H}{\dot{W}_{HP}} \leq \beta_{Carnot \; HP}$

Second law: $\beta_{Carnot \; HP} = \frac{T_H}{T_H - T_L} = \frac{120 + 273.15}{120 - 90} = 13.1$

$$\dot{Q}_H = \dot{Q}_L + \dot{W} \Rightarrow \frac{\dot{Q}_L}{\dot{W}_{HP}} = \beta_{Carnot \; HP} - 1 = 13.1 - 1 = 12.1$$

$$\dot{W} = \dot{Q}_L / (\beta_{Carnot \; HP} - 1) = \frac{5000}{12.1} = 413 \; kW$$

The stated 400 kW is thus not sufficient. Furthermore, we computed the power input requirement assuming an ideal heat pump. Any actual heat pump would have a lower COP and thus a higher required power input (probably 2-3 times as much)

Comment: Instead of using any external work input to this process the waste heat can be divided into two with one fraction used in a heat engine rejecting energy to the ambient providing the work to drive the heat pump for the second part of the waste energy.
Problem-4(20%)
A tray for making ice-cubes is filled with 0.5 kg liquid water at 15°C. It is now put into the freezer where it is cooled down to −10°C using 50 W of electrical power input. The freezer sits in a room at 25°C having a coefficient of performance as \( \beta_{\text{refrigerator}} = \beta_{\text{Carnot ref}} / 3 \). How much energy is used as work input to accomplish this process? If we neglect other cooling loads how much time will this process take?

Solution:

C.V. Water in the tray.
This is a control mass. As the water freezes it happens at constant pressure of 101 kPa.

Energy Eq.: \( m(u_2 - u_1) = 1Q_2 - 1W_2 \)

State 1: Compressed liquid, use saturated liquid same T, from Table B.1.1
\[ u_1 = 62.98 \text{ kJ/kg}, \quad v_1 = 0.001001 \text{ m}^3/\text{kg} \]

State 2: Compressed solid, use saturated solid same T, from Table B.1.5
\[ u_2 = -354.09 \text{ kJ/kg}, \quad v_2 = 0.0010891 \text{ m}^3/\text{kg} \]

Process: \( P = C \);
\[ 1W_2 = P \cdot m(v_2 - v_1) \]
\[ = 101 \text{ kPa} \times 0.5 \text{ kg} \cdot (0.0010891 - 0.001001) \text{ m}^3/\text{kg} = 0.0044 \text{ kJ} \]

From the energy equation
\[ 1Q_2 = m(u_2 - u_1) + 1W_2 = 0.5 (-354.09 - 62.98) + 0.0044 = -208.5 \text{ kJ} \]

Notice how small the work is; it could have been neglected. The heat transfer out of the water is then into the cold air in the freezer which is the \( Q_L \) into the refrigerator cycle.

The Carnot cycle refrigerator has a coefficient of performance COP as, Eq.5.12
\[ \beta_{\text{Carnot ref}} = \frac{T_L}{T_H - T_L} = \frac{273 - 10}{25 - (-10)} = 7.51 \]

and the actual refrigerator has
\[ \beta_{\text{refrigerator}} = \frac{\beta_{\text{Carnot ref}}}{3} = 2.5 \]

Heat transfer out of the water is into the refrigerator cycle, so
\[ W = \frac{-1Q_2}{\beta_{\text{refrigerator}}} = \frac{208.5}{2.5} = 83.4 \text{ kJ} \]

With the constant rate we have \( W = \Delta t \dot{W} \) so the elapsed time is
\[ \Delta t = \frac{W}{\dot{W}} = \frac{83.4 \times 1000 \text{ kJ} \times \text{J/kJ}}{50 \text{ J/s}} = 1668 \text{ s} = 27.8 \text{ min} \]