Steam enters a turbine at 3 MPa, 450°C, expands in a reversible adiabatic process and exhausts at 50 kPa. Changes in kinetic and potential energies between the inlet and the exit of the turbine are small. The power output of the turbine is 800 kW. What is the mass flow rate of steam through the turbine?

Solution:

C.V. Turbine, Steady single inlet and exit flows. Adiabatic: $\dot{Q} = 0$.

Continuity Eq. 4.11: $\dot{m}_i = \dot{m}_e = \dot{m}$.

Energy Eq. 4.12: $\dot{m}h_i = \dot{m}h_e + \dot{W}_T$.

Entropy Eq. 7.8: $\dot{m}s_i + 0 = \dot{m}s_e$ (Reversible $\dot{S}_{gen} = 0$)

Explanation for the work term is in Sect. 7.3, Eq. 7.14

Inlet state: Table B.1.3 $h_i = 3344$ kJ/kg, $s_i = 7.0833$ kJ/kg K

Exit state: $P_e$, $s_e = s_i \Rightarrow$ Table B.1.2 saturated as $s_e < s_g$

$x_e = (7.0833 - 1.091)/6.5029 = 0.92148$,

$h_e = 340.47 + 0.92148 \times 2305.40 = 2464.85$ kJ/kg

$\dot{m} = \dot{W}_T/\dot{W}_T = \dot{W}_T/(h_i - h_e) = \frac{800}{3344 - 2464.85} \frac{kW}{kJ/kg} = 0.91$ kg/s
The exit nozzle in a jet engine receives air at 1200 K, 150 kPa with negligible kinetic energy. The exit pressure is 80 kPa and the process is reversible and adiabatic. Use constant heat capacity at 300 K to find the exit velocity.

Solution:
C.V. Nozzle, Steady single inlet and exit flow, no work or heat transfer.

Energy Eq.4.13: \[ h_i = h_e + \frac{V_e^2}{2} \quad (Z_i = Z_e) \]

Entropy Eq.7.9: \[ s_e = s_i + \int \frac{dq}{T} + s_{gen} = s_i + 0 + 0 \]

Use constant specific heat from Table A.5, \( C_{p0} = 1.004 \frac{kJ}{kg \ K}, \ k = 1.4 \)

The isentropic process \( (s_e = s_i) \) gives Eq.6.23

\[
\Rightarrow \quad T_e = T_i \left( \frac{P_e}{P_i} \right)^{\frac{k-1}{k}} = 1200 K \left( \frac{80}{150} \right)^{0.2857} = 1002.7 K
\]

The energy equation becomes

\[
\frac{V_e^2}{2} = h_i - h_e = C_p(T_i - T_e)
\]

\[
V_e = \sqrt{2 C_p(T_i - T_e)} = \sqrt{2 \times 1.004(1200-1002.7)} \times 1000 = 629.4 \text{ m/s}
\]
A small turbine delivers 1.5 MW and is supplied with steam at 700°C, 2 MPa. The exhaust passes through a heat exchanger where the pressure is 10 kPa and exits as saturated liquid. The turbine is reversible and adiabatic. Find the specific turbine work, and the heat transfer in the heat exchanger.

Solution:

Continuity Eq. 4.11:  Steady
\[ \dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m} \]

Turbine: Energy Eq. 4.13:  \[ w_T = h_1 - h_2 \]

Entropy Eq. 7.9:  \[ s_2 = s_1 + s_{T_{gen}} \]

Inlet state:  Table B.1.3  \[ h_1 = 3917.45 \text{ kJ/kg}, \quad s_1 = 7.9487 \text{ kJ/kg K} \]

Ideal turbine  \[ s_{T_{gen}} = 0, \quad s_2 = s_1 = 7.9487 = s_{t_2} + x_{s_{fg_2}} \]

State 3:  \[ P = 10 \text{ kPa}, \quad s_2 < s_g = \text{saturated 2-phase in Table B.1.2} \]

\[ \Rightarrow x_{2,s} = (s_1 - s_{t_2})/s_{fg_2} = (7.9487 - 0.6492)/7.501 = 0.9731 \]

\[ \Rightarrow h_{2,s} = h_{t_2} + x_{h_{fg_2}} = 191.8 + 0.9731 \times 2392.8 = 2520.35 \text{ kJ/kg} \]

\[ w_{T,s} = h_1 - h_{2,s} = 1397.05 \text{ kJ/kg} \]

\[ \dot{m} = \dot{W} / w_{T,s} = 1500 / 1397 = 1.074 \text{ kg/s} \]

Heat exchanger:

Energy Eq. 4.13:  \[ q = h_3 - h_2 , \]

Entropy Eq. 7.9:  \[ s_3 = s_2 + \int dq/T + s_{He_{gen}} \]

\[ q = h_3 - h_{2,s} = 191.83 - 2520.35 = -2328.5 \text{ kJ/kg} \]

\[ \dot{Q} = \dot{m} q = 1.074 \text{ kg/s} \times (-2328.5) \text{ kJ/kg} = -2500 \text{ kW} \]

Explanation for the work term is in Sect. 7.3, Eq. 7.14