**6.52**

\[ \bar{x} = 10.500 \text{ inches} \]
\[ \sigma = 0.005 \text{ inches} \]

a) \[ z = \frac{10.520 - 10.500}{0.005} = \frac{0.020}{0.005} = 4 \]
\[ P(x \leq 10.520) = 0.5 + 0.5 = 1 \Rightarrow 100\% \text{ U sin g Table 6.3} \]

b) \[ z_1 = \frac{10.485 - 10.500}{0.005} = -3 \]
\[ z_2 = \frac{10.515 - 10.500}{0.005} = 3 \]

From Table 6.3, \( A = 0.4987 \)
\[ P(10.485 < x < 10.515) = 2 \times 0.4987 = 0.9974 \Rightarrow 99.87\% \text{ From Table 6.3} \]

c) From Table 6.3,
\[ \bar{z} = 2.5, P(\bar{x} - 2.5\sigma \leq x \leq \bar{x} + 2.5\sigma) = 2 \times 0.4938 = 0.9876 \Rightarrow 98.76\% \]
\[ P(\text{rejection}) = 1 - 0.9876 = 0.012 \]

**6.54**

a) \( \sigma = 5000 \)
From table 6.3 for area = 0.40 \( z \approx 1.28 \)
\[ \Rightarrow \frac{x - 50,000}{5000} = -1.28 \]
\[ x = 43,600 \text{ miles} \]

b) \[ z_1 = \frac{60,000 - 50,000}{5000} = 2 \quad \text{area} = 0.4772 \]

\[ z_2 = \frac{70,000 - 50,000}{5000} = 4 \quad \text{area} = 0.5 \]
\[ \Rightarrow 0.5 - 0.4772 = 0.0228 \]

\( (100,000 \text{ tires})(0.0228) = 2280 \text{ tires fail between 60,000 to 70,000 miles} \)

c) \[ \frac{20,000 - 50,000}{5000} = -6 \]
\[ \Rightarrow \text{No tires expected to have life less than 20,000} \]

d) Major assumption: Life span of tire follow normal distribution.
Confidence level: 95%

\[ 1 - \alpha = 0.95 \quad \Rightarrow \alpha = 0.05 \]

\[ 0.5 - \frac{\alpha}{2} = 0.475 \quad \Rightarrow z = 1.96 \text{ (Table 6.3)} \]

\[ \mu = \overline{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \]

\[ \mu = 30 \pm \frac{(1.96)(2)}{\sqrt{40}} \]

\[ \mu = 30 \pm 0.620 \text{ mph with confidence of 95%} \]

For a large sample, \( n > 30 \), \( \mu = \overline{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \)

For 90, 95, and 99% confidence levels, \( x_{\frac{\alpha}{2}} = 1.64, 1.96, \text{ and } 2.58 \) respectively.

For 90% confidence level, the confidence interval is \( \pm 1.64 \times 0.2/(40)^{1/2} = \pm 0.052 \text{ oz} \). For 95% and 99% confidence levels, the intervals are \( \pm 0.061 \text{ oz} \) and \( \pm 0.082 \text{ oz} \).

The mean is 16.042 oz. and the sample standard deviation is 0.079 oz. The standard deviation of the mean is \( 0.07941/(12)^{1/2} = 0.0229 \text{ oz} \).

(a) This is a t-distribution problem with \( \nu = 12 - 1 = 11 \) and \( \alpha/2 = 0.025 \). From the t-distribution table, \( t \) has a value of 2.201. The confidence interval on the mean is then \( \pm t\alpha/2S/(n)^{1/2} = 2.201 \times 0.0229 = 0.0504 \text{ oz} \).

Find 95% confidence interval on the mean

\[ \overline{x} = 50,000 \text{ miles} \]

\[ S = 5000 \text{ miles} \]

\[ n = 100 \]

\[ 1 - \alpha = 0.95 \quad \Rightarrow \alpha = 0.05 \]

\[ 0.5 - \frac{\alpha}{2} = 0.475 \quad z = 1.96 \text{ (From Table 6.3)} \]

\[ \mu = \overline{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} \]

\[ \mu = 50,000 \pm 1.96(5000 / \sqrt{100}) \]

\[ \mu = 50,000 \pm 980 \text{ miles} \]