Math 110, Fall 2009, Midterm #1, Solutions

1. The slope: $m = \frac{-4-2}{5-(-3)} = -\frac{3}{4}$. An equation of the line: $y = -\frac{3}{4}x + b$. Using the first point:

$$b = y_1 - mx_1 = 2 - (-\frac{3}{4})(-3) = -\frac{1}{4}.$$

Answer: $y = -\frac{3}{4}\,x - \frac{1}{4}$.

2. We are told that $C = 6000$ when $x = 1000$, and $C = 8500$ when $x = 1500$. We use two points, $(1000, 6000)$ and $(1500, 8500)$, on the graph of the cost function $C = mx + b$:

$$m = \frac{C_2 - C_1}{x_2 - x_1} = \frac{8500 - 6000}{1500 - 1000} = 5,$$

so the marginal cost is \$5 per case of soda. Now

$$b = C_1 - mx_1 = 6000 - 5 \cdot 1000 = 1000,$$

so the fixed cost is \$1000.

Answer: The fixed cost is \$1,000 and the marginal cost is \$5 per case.

3. *Common sense solution:* Because the population doubles every 3 hours, it will be 8 times the original amount in $3 + 3 + 3 = 9$ hours.

Another solution:

We know that

$$2\,A = A\,b^3 \quad \Rightarrow \quad b^3 = 2 \quad \Rightarrow \quad b = \sqrt[3]{2}$$

To find the required time, we need to solve the equation

$$8\,A = A\,b^t$$

for t. We have

$$b^t = 8 \quad \Rightarrow \quad t \cdot \ln b = \ln 8 \quad \Rightarrow \quad t = \frac{\ln 8}{\ln \sqrt[3]{2}} = 9$$

Answer: 9 hours.

4. The balance at Bank A will be (use formula $\mathcal{A}(t) = P(1 + \frac{r}{m})^{mt}$):

$$\mathcal{A}_A = 1000(1 + 0.12)^{10} = \$3,105.85$$

and at Bank B will be (use formula $\mathcal{A}(t) = Pe^{rt}$):

$$\mathcal{A}_B = 1000\,\mathrm{e}^{(0.118)(10)} = \$3,254.37$$

Answer: You should choose Bank B.

5. We use the formula $\mathcal{A}(t) = Pe^{rt}$ to obtain the equation

$$1500 = 900e^{0.08t}.$$

From this equation we have

$$e^{0.08t} = \frac{1500}{900} = \frac{5}{3} \approx 1.667 \quad \Rightarrow \quad 0.08t = \ln 1.667$$

Therefore,

$$t = \frac{\ln 1.667}{0.08} \approx 6.39$$

Answer: 6.39 years