

Finding the Center of an Inscribed Triangle

Given three points defining a triangle, we will construct the center and radius of the inscribed circle. All lines in [Courier New](#) can be entered into TestGiver.

We begin with three points, $\mathbf{u}, \mathbf{v}, \mathbf{w}$, defining a triangle.

```
u=[3;7];v=[7;1];w=[-3;-3];
```

The vector \mathbf{uv} is a unit vector parallel to $\mathbf{v} - \mathbf{u}$, and similarly for \mathbf{uw} and \mathbf{vw} .

```
uv=(v-u)/norm(v-u,2);uw=(w-u)/norm(w-u,2);vw=(w-v)/norm(v-w,2);
```

$\mathbf{u1}$ is a direction vector for the line through \mathbf{u} bisecting the angle at \mathbf{u} . $\mathbf{v1}$ is a direction vector for the line through \mathbf{v} bisecting the angle at \mathbf{v} .

```
u1=uv+uw;  
v1=-uv+vw;
```

The line through \mathbf{u} with direction $\mathbf{u1}$ has equation $ax+by=c$.

The line through \mathbf{v} with direction $\mathbf{v1}$ has equation $dx+ey=f$.

```
a=u1[2];b=-u1[1];c=a*u[1]+b*u[2];  
d=v1[2];e=-v1[1];f=d*v[1]+e*v[2];
```

The two lines intersect at the point $(x1,y1) = \mathbf{cc}$.

```
x1=(c*e-b*f)/(a*e-b*d);  
y1=(a*f-c*d)/(a*e-b*d);
```

```
cc=[x1;y1];
```

\mathbf{cu} is the vector from vertex \mathbf{u} to center \mathbf{cc} .

\mathbf{cv} is the vector from vertex \mathbf{v} to center \mathbf{cc} .

\mathbf{cw} is the vector from vertex \mathbf{w} to center \mathbf{cc} .

```
cu=cc-u;  
cv=cc-v;  
cw=cc-w;
```

\mathbf{pw} is the point where the perpendicular from the center \mathbf{cc} meets the line through \mathbf{u} and \mathbf{v} (since $|\mathbf{uv}|=1$).

```
pw=(cu.uv)*uv+u;
```

The radius of the inscribed circle is the distance from the center \mathbf{cc} to \mathbf{pw} .

`norm(pw-cc, 2)`