Introduction to Probability (Ch. 4)

Techniques for finding the size of the sample space $S$ of a random experiment:
- Counting Rule: multiply the # of outcomes from each step of a multi-step experiment
- Combinations: the number of ways to choose $r$ things from $n$ (order is not important)
- Permutations: the number of ways to arrange $r$ things from $n$ (order is important)

Views of Probability:
1. Relative Frequency: An event $A$’s probability $P(A) = \text{the } \% \text{ of time } A \text{ occurs in many trials. Use when actual data are available.}$
2. Classical or Theoretical: Assume all elementary events in $S$ have equal probability.
   \[ P(A) = \frac{\text{the # of ways } A \text{ can occur}}{\text{the # of elementary outcomes in the sample space}}. \]

Basic Definitions & Laws of Probability:
1. The probability of any event $A$ is a number between 0 and 1:
   \[ 0 \leq P(A) \leq 1 \]
2. When an experiment takes place, some event must occur:
   \[ \sum P(A_i) = 1 \]
3. Law of the Complement: \( P(A^c) = P(\text{"not } A") = 1 - P(A) \)
4. Addition Law: \( P(A \cup B) = P(A \text{ or } B) = P(A) + P(B) - P(A \cap B) \)
5. Events $A$ & $B$ are disjoint if \( P(A \cap B) = P(A \text{ and } B) = P(B \cap A) = 0 \)
6. Conditional probability of $A$ given $B$: \( P(A|B) = \frac{P(A \cap B)}{P(B)} \)
7. Events $A$ & $B$ are statistically independent if \( P(A \cap B) = P(A)P(B), \text{ or if } P(A|B) = P(A) \)
8. Multiplication Law: \( P(A \cap B) = P(A|B)P(B) = P(B|A)P(A) \)
9. Multiplication Law for Independent Events: \( P(A \cap B) = P(A)P(B) \)
10. An interval’s probability: \( P(c \leq X \leq d) = P(X \leq d) - P(X < c) = \text{Diff. of cumulative probs.} \)

Discrete Probability Distributions (Ch. 5)

Random Variable (RV): A variable whose value depends on the outcome of a random event.
- **Discrete RVs** take on a set of distinct values, e.g., the sum of 2 dice, the # heads in 5 flips, etc.
- **Continuous RVs**, by contrast, take on an infinite set of values, e.g., time, height, weight, etc.
- \( f(x) \) is the probability function associated with $X$, i.e., \( f(x) = P(X = x) \)
- The distribution of $X$ can be thought of as the graph of $f(x)$ for all values $x$ that $X$ can take on.
- $X$ has an expected value \( E[X] \) or mean $\mu = \sum x f(x) = \text{the long-run average value of } X$.
- $X$ has variance $\sigma^2 = \sum (x - \mu)^2 f(x)$ and standard deviation $\sigma = +\sqrt{\text{Variance}}$.

**Binomial($n$, $p$) RV:** A special RV that counts the # of successes in $n$ identical “binary” trials when:
1. The number of trials $n$ is fixed;
2. The $n$ trials are statistically independent of one another;
3. Each trial results in either a “success” or “failure”; and
4. The probability of success on each trial $p$ does not change from trial to trial.
   The Binomial probability function is \( f(x) = \binom{n}{x} p^x q^{n-x}, \text{ for } x = 0, 1, \ldots, n. \)

**Poisson($\mu$) RV:** A special RV that counts the # of events in an interval of time (or space) when:
1. The mean number of randomly occurring events in an interval of time (or space) is $\mu$;
2. The probability of an event occurring is the same for any 2 intervals of equal length; and
3. Events occurring in one interval are independent of events occurring in any other interval.
   The Poisson probability function is \( f(x) = \mu^x e^{-\mu}/x!, \text{ for } x = 0, 1, 2, 3, \ldots \)