Recall: Billing Dept. Relocation

To save money, a large company is relocating its billing division from S.F. to Iowa. Can it be done in less than 6 months?

<table>
<thead>
<tr>
<th>Activity Symbol</th>
<th>Activity Description</th>
<th>Immediate Predecessors</th>
<th>Estimated Time (wks.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>Office site selection</td>
<td>–</td>
<td>3</td>
</tr>
<tr>
<td>P</td>
<td>Personnel needs decided</td>
<td>–</td>
<td>4</td>
</tr>
<tr>
<td>D</td>
<td>Design &amp; build facility</td>
<td>O, P</td>
<td>12</td>
</tr>
<tr>
<td>S</td>
<td>Select people to move</td>
<td>P</td>
<td>2</td>
</tr>
<tr>
<td>H</td>
<td>Hire new people in Iowa</td>
<td>S</td>
<td>4</td>
</tr>
<tr>
<td>M</td>
<td>Move SF people to Iowa</td>
<td>D, S</td>
<td>2</td>
</tr>
<tr>
<td>T</td>
<td>Train new people in Iowa</td>
<td>H, M</td>
<td>3</td>
</tr>
</tbody>
</table>

CPM Network: AON

- With a small network, can list all paths
- Critical Path = the longest (slowest) path
Find Critical Path: 2-Pass Method

1. → Forward Pass: Find ES & EF
   ES = Earliest Start time, if all goes well before it;
   EF = Earliest Finish time = ES + Activity Time.

<table>
<thead>
<tr>
<th>ES</th>
<th>EF</th>
<th>Calculate 4 things for each activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>LS</td>
<td>LF</td>
<td></td>
</tr>
</tbody>
</table>

2. ← Backward Pass: Find LF & LS
   LF = Latest Finish time without delaying activities that follow it;
   LS = Latest Start time = LF – Activity Time

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Why bother? Want to get each activity’s slack = LF – EF = LS – ES

EX: Activity O: 

\[
\begin{array}{cc}
\text{ES} &= 0 \\
\text{LS} &= 1 \\
\text{EF} &= 3 \\
\text{LF} &= 4 \\
\end{array}
\]

… has 1 wk. slack

<table>
<thead>
<tr>
<th>ES</th>
<th>LS</th>
<th>EF</th>
<th>LF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

- As long as O starts between 0 & 1, it will finish between 3 & 4, and the project won’t be delayed
- Delaying O by more than 1 week will delay project

Slack Time

- Slack = Amount of time activity can be delayed without delaying the project
- Critical Activities have 0 slack
- Critical Path = The set of critical activities
  - the longest path in the network
- What if P were delayed 5 weeks?
- What if H were delayed 6 weeks? 10 weeks?
Ways to Reduce Project Time

1. Shift resources from non-critical to critical tasks
   EX: Suppose can ↓ M’s time from 2 wks. to 1 wk.
   at the expense of ↑ H’s time from 4 wks. to 5 wks.
   What impact would this have?

2. Add resources to speed up a critical activity
   $$$: Hire more people, or buy faster equipment

3. Do more activities in parallel
   $$$: Subcontract out some tasks

PERT

Main differences with CPM:
1. Activities are drawn on the arcs: AOA Network
   – Looks different but has some advantages
2. Activity times are viewed as random variables
   – Manager estimates each activity with 3 points:
     • $a =$ optimistic time  } Based on
     • $m =$ most-likely time   } the Beta
     • $b =$ pessimistic time   } distribution
Probabilistic Activity Time

EX: Activity P

- Need not be symmetric
- Estimated mean \( t = (a + 4m + b)/6 = 24/6 = 4 \)
- Estimated variance \( \sigma^2 = [(b - a)/6]^2 = [8/6]^2 \)

Relocation Project in PERT

*Suppose project manager comes up with:*

<table>
<thead>
<tr>
<th>Activity Symbol</th>
<th>Immediate Predecessors</th>
<th>Estimated times a m b</th>
<th>Calculated t</th>
<th>( \sigma^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>–</td>
<td>2 2.5 6</td>
<td>3</td>
<td>0.44</td>
</tr>
<tr>
<td>P</td>
<td>–</td>
<td>2 3 10</td>
<td>4</td>
<td>1.78</td>
</tr>
<tr>
<td>D</td>
<td>O, P</td>
<td>9 11 19</td>
<td>12</td>
<td>2.78</td>
</tr>
<tr>
<td>S</td>
<td>P</td>
<td>1.5 2 2.5</td>
<td>2</td>
<td>0.03</td>
</tr>
<tr>
<td>H</td>
<td>S</td>
<td>2 4 6</td>
<td>4</td>
<td>0.44</td>
</tr>
<tr>
<td>M</td>
<td>D, S</td>
<td>2 2 2</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>T</td>
<td>H, M</td>
<td>1.5 3 4.5</td>
<td>3</td>
<td>0.25</td>
</tr>
</tbody>
</table>
PERT Network: AOA

- Nodes make beginning & ending of activities

- With practice, the time scale can be made accurate

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PERT

- Find critical path as before with CPM:
  (1) List-all-paths, or
  (2) 2-pass method
- Get same critical path as before: \{P, D, M, T\}
- But now we can make probability statements about the project’s total time based on the critical path’s mean & variance …

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View Total Project Time (PT) as a RV

- Assume PT is \( \approx \text{Normal}(\mu_{cp}, \sigma_{cp}) \)
  - This is reasonable because of the Central Limit Theorem.
  - \( \mu_{cp} = \text{Mean time along the critical path} \)
    - Here, \( \mu_{cp} = t_p + t_D + t_M + t_T = 21 \) weeks
  - \( \sigma^2_{cp} = \text{Variance along the critical path} \)
    - Variance \( \sigma^2_{cp} = \sigma^2_p + \sigma^2_D + \sigma^2_M + \sigma^2_T = 4.81 \) weeks\(^2\)
    - Therefore, \( \sigma_{cp} = 2.19 \) weeks
- Now can answer probability questions about PT

Project Time Probability Questions

- \( P\{PT \leq 21 \text{ weeks}\} = ? \)
- \( P\{PT \leq 24 \text{ weeks}\} = ? \)
- \( P\{PT > 24 \text{ weeks}\} = ? \)
- \( P\{22 \text{ wks.} \leq PT \leq 24 \text{ wks.}\} = ? \)
- \( P\{PT \leq 20 \text{ weeks}\} = ? \)