THE FORMULATIONS OF SHEAR FORCE AND OVERTURNING MOMENT OF
THE LARGE-UPRIGHT UNANCHORED INDUSTRIAL LIQUID STORAGE TANKS
SUBJECTED TO HORIZONTAL GROUND EXCITATIONS

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SUMMARY

The dynamic response analysis is performed for the formulations of shear force and overturning moment of the large-upright unanchored industrial liquid storage tanks subjected to horizontal ground acceleration. As the tank is accelerated in the horizontal direction, it tends to uplift from its foundation, and hydrodynamic pressures on the tank wall vary with height in non-linear fashion. In this study, the distribution of hydrodynamic pressures and its center are directly correlated to formulate shear force and overturning moment. Initially, the equations of shear force and overturning moment are derived by assuming hydrodynamic pressures exerted on tank wall would vary in parabolic trend. Then derived equations are multiplied by dynamic coefficients, which are basically the function of peak ground acceleration, excitation frequency and the ratio of liquid’s height to radius of tanks. Dynamic coefficients are formulated through the shake table experiment of the model tanks excited by computer generated ground motion. The equations proposed in this paper for base shear and overturning moment are only the function of total weight of tank, the ratio of liquid’s height to radius, specific weight of liquid and dynamic coefficients for shear force and overturning moment. Therefore, the proposed equations are very simple, efficient and easy to perform in calculating of shear forces and overturning moments of the large-upright industrial liquid storage tanks subjected to lateral earthquake loads. The results are verified with different codes (e.g. Eurocode8, API and AWWA-100...).

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INTRODUCTION

The liquid content of the tank subjected to horizontal ground excitations demonstrate two-type of responses: Impulsive response and convective - sloshing- response.

When the tank is excited in horizontal direction, the total linear momentum of the system changes, and the time rate of the linear momentum becomes total shear force. The main component of shear force is coming from impulsive and convective response of the liquid content of the tank. The fraction of the mass of the total liquid vibrates at the frequency of the rigid container is called impulsive mass. The fraction of the mass of the liquid that vibrates its own frequency is called convective mass.

Due to impulsive and convective response of the liquid, hydrodynamic pressures exert on the tank's wall as seen in Fig. 1. The hydrodynamic pressures vary with the height of the tank during horizontal excitation. The resultant force due to hydrodynamic response of the liquid constitutes the shear force. And the center of the hydrodynamic pressure becomes the center of the seismic forces. The moment of the resultant seismic force respect to base becomes overturning moment of the tank.

When the tank diameter decreases, the fraction of the mass of the total liquid that vibrates at the frequency of the rigid container gets larger. As a result, the resultant force due to impulsive mass increases. In this case impulsive response is dominant factor for the seismic forces exert on tank body. On the other hand, when the tank diameter gets larger, impulsive mass of the liquid decreases and the convective mass increases respectively. As a result, convective response becomes a major factor for the amplitude of the seismic forces. The design seismic coefficient of impulsive mode is much higher than the design seismic coefficient of convective mode. As a result shear force decreases significantly when convective mass gets larger.
The common procedures in calculating shear force and overturning moment are basically to quantify dynamic properties of tank subjected to horizontal excitation. Those properties are mainly as follows: Impulsive and convective periods, impulsive and convective masses, impulsive and convective heights, impulsive and convective- spectral accelerations, impulsive and convective coefficients and so on...

Primarily, engineers follow the procedures described by different standards or codes to quantify dynamic properties required in calculating shear force end overturning moment of the tank when subjected to horizontal excitations. But those procedures are generally confusing and difficult to apply. They are also frequently modified without demonstrating proper rationales and used by different equations for same properties and designed with different spectral acceleration, tables and coefficients...

For simplification purposes, the main focus of this study are to formulate shear force and overturning moment of the tank without quantifying dynamic properties. In this study, hydrodynamic pressures exerted on the tank wall and dynamic coefficients for shear force and overturning moment are the fundamental parameters. Dynamic coefficients of shear force and overturning moment are obtained by shake table experiment for model tanks. The key questions answered in this study are as follows: How would the hydrodynamic pressures be mathematically correlated to shear force and overturning moment? Where is the center of the hydrodynamic pressures, which is also the center of the seismic forces?

**Procedure**
In this study, the equations of shear force and overturning moment are initially derived by assuming that the hydrodynamic pressures vary with height in parabolic trend as shown in Fig. 2. Where, \( y \) is the elevation of a point on the shell measured from the base, \( H \) is the fluid depth, \( p_0 \) is the static pressure amplitude at the tank base and \( p \) is hydrodynamic pressure due to horizontal excitation of the tank.

![Fig. 2. The kinetic diagram and the variation of hydrodynamic pressure with the height of the liquid in tank](image-url)
The derived equations are then multiplied with the factors called dynamic coefficients, which are obtained through computer generated base excitation on the models of the liquid storage tanks.

The schedules of the tank models are shown in table 1.

<table>
<thead>
<tr>
<th>Tank Schedule #</th>
<th>Shell Thickness (mm)</th>
<th>W (ton.f)</th>
<th>Weight of Liquid Content</th>
<th>H/R</th>
<th>R (m)</th>
<th>H (m)</th>
<th>1.20H (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.50</td>
<td>0.10</td>
<td>0.5</td>
<td>0.40</td>
<td>0.20</td>
<td>0.24</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.40</td>
<td>0.10</td>
<td>1.0</td>
<td>0.32</td>
<td>0.32</td>
<td>0.38</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0.35</td>
<td>0.10</td>
<td>1.5</td>
<td>0.28</td>
<td>0.42</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0.31</td>
<td>0.10</td>
<td>2.0</td>
<td>0.25</td>
<td>0.50</td>
<td>0.60</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.29</td>
<td>0.10</td>
<td>2.5</td>
<td>0.23</td>
<td>0.58</td>
<td>0.70</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.27</td>
<td>0.10</td>
<td>3.0</td>
<td>0.22</td>
<td>0.66</td>
<td>0.79</td>
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</tr>
</tbody>
</table>

The schematic of the experimental setup is illustrated in Fig. 3. The models are excited with the harmonic ground excitation, which the excitation frequency is \( \omega = 3\pi \text{ rad/s} \), and peak ground acceleration is \( 1.0^*g \). Hydrodynamic pressures are measured from the bottom at every \( H/4 \) of the height of the tank. The load cell also measures the actual shear force. The related data collected by the data acquisition system (DAS) at the sampling rate of 60 readings/s.

Fig. 3. The schematic of the experimental setup of the model tanks connected to DAS
**Formulations:**

Base shear is the total lateral force exerts on the base, which is correlated to the hydrodynamic pressures, \( p \), and the dynamic coefficient of the shear force, \( k_1 \). Overturning moment, \( M \) is the moment of the seismic force respect to base. Overturning moment is also the function of the hydrodynamic pressure, \( p \) and dynamic coefficient for overturning moment, \( k_2 \). Base shear, \( Q \) and overturning moment, \( M \) can be formulated as follows:

\[
Q = k_1 \int dQ ; \quad dQ = k_1 p dA \\
M = k_2 \int dM ; \quad dM = k_2 y dQ ; \quad M = \bar{y} Q
\]

Where;

\( Q \) ....Base shear  

\( M \) ....Overturning moment  

\( k_1 \) ....Dynamic Coefficient of shear force (Fig. 4)(dimensionless)  

\( k_2 \) ....Dynamic Coefficient of overturning moment (Fig. 4)(dimensionless)  

\( p_0 \) ....Hydrostatic pressure at base \( p_0 = \gamma H \)  

\( p \) ....Hydrodynamic pressure as a function of height \( p = p_0 (1 - \frac{y^2}{H^2}) \) (assumed)  

\( \bar{y} \) ....The center of the hydrodynamic pressures

Shear force, \( Q \), would be derived as a function of \( k_1, (\frac{H}{R}) \) and \( W \) as follows:

\[
Q = k_1 \int p dA = k_1 \int_0^H 2 R p_0 (1 - \frac{y^2}{H^2}) dy = k_1 (\frac{4}{3}) H p_0 R = k_1 (\frac{4}{3}) H^2 \gamma R
\]

\[
Q = k_1 (\frac{4}{3}) (\frac{H}{R})^2 R^3 \gamma , \quad \text{Substituting} \quad R = (\frac{1}{\gamma \sqrt{\pi}}) \frac{W}{\sqrt{\pi} \gamma S \left( \frac{H}{R} \right)^{-1}}
\]

\[
Q = \frac{4}{3} (k_1) (\frac{H}{R}) (\frac{W}{\pi})
\]

\[ (1) \]
Overturning moment, M would be derived as a function of $k_2 \left( \frac{H}{R} \right), W$ as follows:

$$M = k_2 \int_0^H pydA = k_2 \int_0^H 2Rp_0 \left(1 - \frac{y^2}{H^2} \right) y dy = k_2 \frac{2}{4} H^2 \rho_o R = k_2 \frac{1}{2} H^3 \gamma R$$

$$M = k_2 \frac{1}{2} \left( \frac{H}{R} \right)^3 R^4 \gamma \quad \text{Substituting } R = \left( \frac{W}{\pi \gamma S} \right)^{-1}$$

$$M = \frac{1}{2} (k_2) \left( \frac{H}{R} \right)^{(5/3)} (\gamma)^{(-1/3)} \left( \frac{W}{\pi} \right)^{(4/3)} \quad (2)$$

Where:

$H ...$. Maximum height of liquid in tank

$R ...$. Radius of the tank ($R = \left( \frac{W}{\pi \gamma S} \right)^{-1}$)

$W ...$. Total weight of liquid in tank ($W = \gamma \pi R^2 H$)

$A ...$. The cross section area of the tank perpendicular to horizontal direction ($A = \int dA = \int (2R)dy$)

$S ...$. Specific gravity of liquid; ($S_{water} = 1$)

$\gamma ...$. Specific weight of liquid ($\gamma = \gamma_w S$)

$\gamma_w ...$. Specific weight of pure water (1000kgf/m³, or 62.4Lb/ft³)

The model experimentations yield to quantify the actual shear force, $Q_{actual}$, the center of the hydrodynamic pressures, $\bar{y}$ and corresponding actual overturning moments, $M_{actual}$ of the model tanks. The results are tabulated in Table. 2 and in Table. 3.

$k_1$ and $k_2$ are dynamic coefficients of base shear and overturning moment, which are $k_1 = \frac{Q_{act}}{Q}$ and $k_2 = \frac{M_{act}}{M}$. The $k_1$ and $k_2$ vs. (H/R) are illustrated in Fig. 4.
Table 2: Parameters of the dynamic coefficient, $k_1$ for shear force

<table>
<thead>
<tr>
<th>Tank Schedule #</th>
<th>H/R (m)</th>
<th>H (m)</th>
<th>R (m)</th>
<th>$(Q_{actual})$ Shear Force KN</th>
<th>$Q = \frac{4}{3}(k_1)\left(\frac{H}{R}\right)\left(\frac{W}{\pi}\right)$ WHEN $(k_1 = 1)$</th>
<th>$k_1 = \frac{Q_{act}}{Q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.20</td>
<td>0.40</td>
<td>0.175</td>
<td>0.208</td>
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<td>0.32</td>
<td>0.286</td>
<td>0.416</td>
<td>0.686</td>
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<tr>
<td>3</td>
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<td>0.42</td>
<td>0.28</td>
<td>0.349</td>
<td>0.625</td>
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<tr>
<td>4</td>
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<td>0.400</td>
<td>1.041</td>
<td>0.384</td>
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<tr>
<td>6</td>
<td>3.0</td>
<td>0.66</td>
<td>0.22</td>
<td>0.415</td>
<td>1.249</td>
<td>0.332</td>
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</tbody>
</table>

Table 3: Parameters of the dynamic coefficient, $k_2$ for overturning moments

<table>
<thead>
<tr>
<th>Tank Schedule #</th>
<th>H/R (m)</th>
<th>$\gamma$ (m)</th>
<th>$(M_{actual})$ Over-turning Moment KN.m</th>
<th>$M = \frac{1}{2}(k_2)\left(\frac{H}{R}\right)^{5/3}\left(\gamma\right)^{1/3}\left(\frac{W}{\pi}\right)^{4/3}$ WHEN $k_2 = 1$ KN.m</th>
<th>$k_2 = \frac{M_{act}}{M}$</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>0.5</td>
<td>0.09</td>
<td>0.016</td>
<td>0.016</td>
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<td>0.108</td>
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<td>0.474</td>
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<tr>
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<td>3.0</td>
<td>0.31</td>
<td>0.129</td>
<td>0.309</td>
<td>0.417</td>
</tr>
</tbody>
</table>

Dynamic Coefficients of Tanks for Shear Force ($k_1$) and for Overturning Moment ($k_2$)

$\begin{align*}
    k_2 &= 0.0641(H/R)^2 - 0.4582(H/R) + 1.2169 \\
    k_1 &= 0.0523(H/R)^2 - 0.3859(H/R) + 1.0205
\end{align*}$

Fig. 4. Dynamic coefficients $k_1$ and $k_2$ as a function of (H/R) ratios of tanks
Parameter study
Equation (1) and equation(2) are used to calculate shear forces and overturning moments of the large industrial liquid storage tanks. The quantified results for shear forces and overturning moments for different size of tanks are illustrated as mat lab figures in Fig 5.

The sample calculations are also performed with different standards and codes for verification purposes of the proposed equations in calculating shear force and overturning moment. The results of the sample calculations are tabulated in table 4.

Sample problem:
A steel tank with a radius, R of 8 m (D=16m) and total height of 9.6(Ht=9.6) m is stands as unanchored over a thick concrete mat foundation. The tank is filled with water to a height, H of 8 m. The total mass of water in the tank (m_l) is 1.625*10^6 kg (W_l=1625tonf). The uniform thickness of the tank is t= 1.0 cm (t=0.01m). The total mass of the tank wall (m_w) is 40*10^3 kg (W_s=40 tonf), and the mass of the tank roof (m_r) is 1.5*10^3kg (W_r=1.5 tonf). Use 0.5% and 2% damped elastic response spectra for the site.

<table>
<thead>
<tr>
<th>Table. 4 The Results Of Sample Problem Obtained By Proposed Equations And By Different Standards</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposed method and different standards</td>
</tr>
<tr>
<td>-----------------------------------------------</td>
</tr>
<tr>
<td>Proposed Method</td>
</tr>
<tr>
<td>Eurocode 8</td>
</tr>
<tr>
<td>API-650,AWWA D100</td>
</tr>
<tr>
<td>IITH-DSDMA Guidelines for Seismic Design of Liquid Storage Tanks</td>
</tr>
</tbody>
</table>
The Base Shears and the Overturning Moments of the Tanks

Fig. 5. The Mat Lab figures of shear forces and overturning moments of the tanks obtained by proposed equations.
CONCLUSION
As seen in Table. 4 the results of the solution of the sample problem by different standards are very consistent with the results of the solution of the proposed equations for shear force and overturning moment of the tank subjected to horizontal excitation. Although dynamic coefficients of shear force and overturning moment acquired in this study based on the certain conditions of horizontal excitation, e.g. $w=3\pi$ rad/s, and peak ground acceleration is $1.0g$, the equation (1) for the shear force and the equation (2) for the overturning moment are the core equations of any types of horizontal excitations. The excitation frequency, peak ground acceleration or excitation type can be different than those that are used in this study. However, the dominant frequency of the most common earthquake is approximately $w=3\pi$ rad/s, or period, $T=0.7$ s., and maximum spectral acceleration is about $0.1g$. Therefore, dynamic coefficients defined in this study would be used to obtain approximate results of shear forces and overturning moments of the tanks subjected to any type of major earthquake motions.

As a result of this study, It is recommended that engineers can perform equation (1) and equation (2) to obtain reliable and quick results in calculating shear forces and overturning moments of the large-upright unanchored industrial liquid storage tanks subjected to lateral earthquake load in preliminary design level by avoiding long and some times confusing procedures required with different standards and codes.

ACKNOWLEDGEMENT
I would like to thank to my dear friend Michael Strange who worked tirelessly to manufacture the model tanks. His contribution to this paper is acknowledged not because he is always a great facilitator, he has broad knowledge and an effective consultant as well.

REFERENCES