EXPERIMENT NUMBER 8

USE OF THE FOURIER TRANSFORM
FOR THE ANALYSIS OF STRUCTURAL VIBRATIONS

OBJECTIVE

The objective of this experiment is to demonstrate the use of the Fast Fourier Transform (FFT) to spectrally decompose the signal from a vibration experiment. The results of the FFT can be used to determine resonant frequencies in a structure which might lead to structural failures in operation.

REFERENCES


Engr. 305 text


INTRODUCTION

A structural element that has both mass and flexibility will have at least one natural frequency. A simple example is a simple spring with a suspended mass as shown in Figure 1. If the mass is displaced a small amount, the mass will continue to oscillate at a frequency known as the resonant frequency if the damping is not too large. For this case with no damping, the natural frequency is found to be:

\[
\omega_n = \left( \frac{k}{m} \right)^{1/2}
\]

If a sinusoidal forcing function with this frequency \([F(t)]\) is applied to the mass, the displacement of the mass \([y(t)]\) will increase in time and may eventually reach a value which is destructive to the system.

A complex structure such as a bridge or aircraft will have many resonant frequencies each with the potential for destructive vibrations. It is usual in design of complex structures to make sure that resonant frequencies are not the same as the frequencies any expected forcing functions. In buildings, one of the main sources of forcing functions is an earthquake. In machines, it might be an engine, a motor or fluid motion. In many cases, resonant frequencies can be determined with structural analysis but in others, it is better to perform a vibration test. In such a test, a forcing function (often of
variable frequency) is applied to the structure and the response of the structure is measured with strain gages or accelerometers. In some cases, the simple expedient of subjecting the structure to a simple displacement or impact will excite a large number of resonant frequencies. It then becomes a matter of analyzing the signal from the sensors (strain gages or accelerometers) to determine resonant frequencies.

The Fast Fourier Transform is a useful mathematical tool for this purpose. A brief introduction to the concept of the Fourier transform is provided in the Engr. 300 text and a more extensive discussion is contained in the Engr. 305 text. Fourier analysis is based on the concept that a complicated time varying signal can be considered to be made up of a set of simple sinusoids, each of different frequency called frequency components. For example, the signal might be equivalent to the sum of sinusoids with frequencies of 1 Hz, 2 Hz, and 3 Hz etc. There are devices called Fourier analyzers which are specifically designed for this purpose. These devices take the output from the sensor and determine the amplitude of these component frequencies. In this experiment, we will record complex signals from a simple structure and analyze the signal using the FFT feature found in common spreadsheet programs. This will enable us to produce a plot such as is shown in Figure 2. In this case, it can be observed that there are high amplitude components at 10 and 15 Hz and small amplitudes at other frequencies.

In this experiment, it will not be our purpose to study the details of Fourier analysis but simply to apply the method to a simple problem.

THE EXPERIMENTAL APPARATUS

The experimental apparatus for this experiment is relatively simple - it consists of a cantilever beam attached to a bench as shown in Figure 3. Four strain gages are attached to the beam near the point at which it is attached to the bench - two on the top and two on the bottom. The strain gages are arranged in a full Wheatstone bridge (all four arms are active gages) and a 6 V battery is used as a power supply. The output is connected to a computer data acquisition system.

There are several methods to excite vibrations in this beam. Some examples are:
1. Vertical displacement of the beam the free end. 
2. Vertical upward displacement at the center
3. Vertical upward displacement at the center and simultaneous downward displacement at the free end (to form an S shape)
4. Striking the center of the beam with a hammer.

Each of these methods of initiating the vibration will excite different Fourier components which can be detected with subsequent analysis with the FFT.

REQUIRED TEST DATA

First measure the beam. Find the height (small dimension), width and the length from the support to the free end.

Connect the output of the Wheatstone bridge to channel zero of a data acquisition system. Connect the six volt power supply to the Wheatstone bridge. Start the data acquisition system and from the DAS subdirectory, start the program SR. Select a gain of 1000. Select a sampling rate of 500 samples per second and 2048 samples. Excite the beam as for condition I above and immediately start taking data. Note: do not start the data acquisition process until your hands have stopped touching the beam. Take additional data for the other initial conditions.

ANALYSIS OF THE DATA

The process of performing an FFT analysis on a set of data is presented in Appendix A-2 of the Wheeler and Ganji text. You are to perform this analysis for each of the cases for which data was taken. For each case, prepare a plot similar to Figure 2 above.

The vibrating cantilever beam is a fairly standard vibration problem and is discussed in many texts.

The beam can actually vibrate in more than one mode. The first two modes and the corresponding natural frequencies are shown in Figure 4 which is taken from an older text by J. P. Den Hartog: Mechanical Vibrations, McGraw-Hill, 1956. Each of the modes can occur simultaneously, the relative magnitudes depending on the initial conditions. In the formulas, $E$ is the modulus of elasticity of the beam material, $I$ is the section modulus, and $\mu$ is the mass per unit length of the beam. Steidel (see references) also analyzes this problem and gives some higher vibrational modes. In the series of which the first two constants in Figure 4 are 3.52 and 22.0, the next three constants are 61.7, 120.91 and 199.85.

![Figure 4 - First two modes of a cantilever beam](image-url)