6.52

\[ n = 15 \]
\[ \bar{x} = 25 \text{ ppm} \quad \text{Find the 95% confidence interval.} \]
\[ S = 3 \text{ ppm} \]

\[ \alpha = 0.05 \quad \nu = n - 1 = 14 \]
\[ \frac{\alpha}{2} = 0.025 \]

\[ \Rightarrow \chi^2_{14,0.025} = 26.119 \quad \text{(From Table 6.7)} \]
\[ \Rightarrow \chi^2_{14,1-0.025} = 5.6287 \]

\[ \frac{(n - 1)S^2}{\chi^2_{14,0.025}} = \frac{14 \times 3^2}{26.119} = 4.82; \quad \frac{(n - 1)S^2}{\chi^2_{14,1-0.025}} = \frac{14 \times 3^2}{5.6287} = 22.17 \]

\[ 4.82 < \sigma^2 < 22.17 \text{ or } 2.20 < \sigma < 4.71 \text{ for 95% confidence level} \]

6.54

Chi squared distribution.
\[ S = 5500, \ n = 8, \ \nu = 7. \ \alpha = 1-0.99 = 0.01, \ \alpha/2 = 0.005, \ 1-\alpha/2 = 0.995. \]
\[ \chi^2_{\alpha/2} = 20.278, \ \chi^2_{1-\alpha/2} = 0.9893. \]

\[ \frac{(n-1)S^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{1-\alpha/2}} \]
\[ \frac{(8-1)5500^2}{20.278} \leq \sigma^2 \leq \frac{(8-1)5500^2}{0.9893} \]

\[ 10442351 \leq \sigma^2 \leq 214.0 \times 10^8 \quad \text{or} \quad 3231 \leq \sigma \leq 14360 \]

6.57 There is an error in the problem. The required variance should be 0.0004 mm². (not m²).

(a) Chi squared distribution.
\[ S = 0.01, \ n = 10, \ \nu = 9. \ \alpha = 1-0.99 = 0.01, \ \alpha/2 = 0.005, \ 1-\alpha/2 = 0.995. \]
\[ \chi^2_{\alpha/2} = 23.589, \ \chi^2_{1-\alpha/2} = 1.7349. \]

\[ \frac{(n-1)S^2}{\chi^2_{\alpha/2}} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{1-\alpha/2}} \]
\[ \frac{(10-1)0.01^2}{23.589} \leq \sigma^2 \leq \frac{(10-1)0.01^2}{1.7349} \]

\[ 3.81 \times 10^{-4} \leq \sigma^2 \leq 5.188 \times 10^{-4} \]

(b) The maximum of the confidence interval is greater than the desired variance so the part is not acceptable.
Data from problem one arranged in ascending order:
48.9, 49.2, 49.2, 49.3, 49.3, 49.8, 49.9, 50.1, 50.2, 50.5

\( \bar{x} = 49.64 \)

\( S = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = 0.530 \)

\( \delta = |x_i - \bar{x}| \)

Low point \( |48.9 - 49.64| = 0.74 \)

High point \( |50.5 - 49.64| = 0.86 \)

From the Table 6.8:
\( n = 10 \Rightarrow \tau = 1.798 \)

\( S\tau = (0.530)(1.798) = 0.95294 \)

Since \( S\tau > \) deviations, then:
No data should be rejected.

From problem 6.4:
\( \bar{x} = 105.2 \)

\( S = 9.71 \)

\( n = 12 \Rightarrow \tau = 1.829 \) (Table 6.8)

\( \delta_1 = |P_{\text{largest}} - \bar{P}| = |120 - 105.2| = 14.8 \)

\( \delta_2 = |P_{\text{smallest}} - \bar{P}| = |89 - 105.2| = 16.2 \)

\( S\tau = (9.71)(1.829) = 17.76 \)

Neither \( \delta_1 \) or \( \delta_2 \) exceeds \( S\tau \) so No points rejected.