Experiment #3: KIRCHHOFF’S LAWS

Objective

To verify experimentally Kirchhoff’s voltage and current laws as well as the principles of voltage and current division.

Apparatus

In this experiment, we use the following apparatus:
- Agilent E3630A Triple Output Power Supply,
- Agilent 34401A Digital Multimeter,
- 10KΩ Potentiometer, and
- a Decade Box.

The Triple Output Power Supply and the Digital Multimeter were described in Lab Report – Experiment #2. Here we will describe the potentiometer and the decade box.

The potentiometer available in our laboratory kit is Bourns 3352. This is a 3/8” single-turn round trimming potentiometer (Figure #1a).

Figure #1. Types of Trimmer Potentiometers

a. muRata PVC6E - Lead Sealed Single-turn Potentiometer
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b. Bourns 3352 - 3/8’’ Single-turn thumb adjustable

A potentiometer, (called “pot” for short), is a resistor with three terminals as shown in Figure # 2. The resistance between terminals A and B is fixed.

![Potentiometer and its equivalent circuit](image)

In our case, this resistance is 10 kΩ. The center terminal is connected to a wiper blade (W) which can be rotated by rotating the shaft. Thus, the resistance between A and W or W and B can be changed by rotating the shaft.

It is obvious that the resistance between W and B is equal to the total resistance minus the resistance between A and W. Thus, any configuration of a pot setting can be represented with two resistors. An example is given in Figure # 3.

![Equivalent circuit for the 10 kΩ potentiometer](image)

A potentiometer can be used as a variable voltage divider or as a variable resistor. In the former case, all three terminals are used (e.g., Figure 5). In the latter case, only two terminals, W and either A or B, are used although it is a
common practice to connect the unused terminal to the wiper to eliminate possible noise pickup.

A decade box is used for accurate resistance reading. The one available in our lab has 6 decade switches to allow for 1,000,000 resistance values to be dialed.

![Decade Box Image]

Figure #4, The TEGAM Model DB62-XX is a DEKABOX in-line Decade Resistor.

### Procedures, Data, Results and Analysis

#### A. Voltage Division

1. Construct the voltage divider circuit (see Fig. 5) with $R_1=1\, \text{k}\Omega$ and $R_2=1\, \text{k}\Omega$.

   We selected two 1\, k\Omega resistors with 5\% tolerance (gold band).

   \[ V_0 = V_1 \frac{R_2}{R_1 + R_2} \]

   ![Voltage Divider Diagram]

   Figure #5. Voltage Divider

2. Turn the power supply on. With the help of the DC voltmeter, adjust the power supply to output +10 V.

   We adjusted the power supply to output 10.0114 V.
3. Measure the voltage around the loop and verify the validity of KVL for this particular circuit. Repeat the loop measurements but with test leads reversed. Do these new readings still satisfy KVL?

Kirchhoff’s Voltage Law (KVL) states that the sum of all voltages around a closed loop is zero.

The voltage measurements are:
- across R1: 5.0079 V
- across R2: 5.0058 V
- across source: -10.0118 V

The sum of all these voltages is zero (within 5% tolerance). Therefore, KVL is satisfied.

If we reverse the test leads, we get the following readings:
- across R1: -5.0078 V
- across R2: -5.0058 V
- across source: 10.0114 V.

Again, since the sum of all voltage changes is zero (within 5% tolerance), KVL is satisfied.

4. Compare the measured $V_o$ with the theoretically computed $V_o$. Account for causes of possible discrepancy.

We compute the value for $V_o$ as follows:

$$V_o = V_i \frac{R_2}{R_1 + R_2} = \frac{10.0114 \text{ V} \times 1 \text{k} \Omega}{1 \text{k} \Omega + 5 \text{k} \Omega} = 5.0057 \text{ V}$$

The measured value for $V_o$ is 5.0058 V. The discrepancy is 0.0001 V, a very small value, within 5% tolerance. Possible cause of discrepancy is some resistance in the wires.

5. Leave R2 in place, repeat steps 3 and 4 for the case $R_1=100 \Omega$ and the case of $R_1=10k \Omega$.

CASE 1: $R_1=100 \Omega$, $R_2=1k \Omega$

The voltage measurements are:
- across R1: 0.9168 V
- across R2: 9.093 V
- across source: -10.0118 V

Sum = - 0.002 V
The sum of all these voltages is close to zero (within 5% tolerance). Therefore, KVL is satisfied.

If we reverse the test leads, we get the following readings:

across R1: -0.9169 V
across R2: -9.0935 V
across source: 10.0114 V

Sum = 0.001 V

Again, since the sum of all voltage changes is close to zero, KVL is satisfied.

The theoretical value for $V$ is:

\[
V = V_1 \frac{R_2}{R_1 + R_2} = 10.0114 \times \frac{0.1 \, \text{k} \Omega}{0.1 \, \text{k} \Omega + 1 \, \text{k} \Omega} = 0.9101 \, \text{V}
\]

The measured value for $V_0$ is 0.9168 V. The discrepancy is 0.0067 V (0.7% error, within 5% tolerance) possibly due to some resistance in the wires and manufacturer's tolerance.

CASE 2: $R_1=10\,\text{k} \Omega$, $R_2=1\,\text{k} \Omega$

The voltage measurements are:

across R1: 99.455 mV = 0.0995 V
across R2: 9.9064 V
across source: -10.0118 V

Sum = -0.0059 V

The sum of all these voltages is close to zero (within 5% tolerance). Therefore, KVL is satisfied.

If we reverse the test leads, we get the following readings:

across R1: -99.457 mV = -0.0995 V
across R2: -9.9035 V
across source: 10.0114 V

Sum = 0.0084 V

Again, since the sum of all voltage changes is close to zero (discrepancy in 1%, within 5% tolerance), KVL is satisfied.

The theoretical value for $V_0$ is:

\[
V_0 = V_1 \frac{R_2}{R_1 + R_2} = 10.0114 \times \frac{10 \, \text{k} \Omega}{10 \, \text{k} \Omega + 1 \, \text{k} \Omega} = 9.1012 \, \text{V}
\]
The measured value for $V_0$ is 9.9064 V. The discrepancy is 0.8052 V, about 8% error from the theoretical value, which is more than the 5% tolerance. It is possible that we made some mistake in the measurements, so...

6. Connect the 10kΩ potentiometer to the power supply as shown below, Fig#6.

![Figure #6. A Variable Voltage Divider](image_url)

7. Monitor $V_o$ with the dc voltmeter as you rotate the pot shaft back and forth. Within what limits does $V_o$ vary? Why?

$V_0$ varies within $V_{\text{max}} = 10.0043 \text{ V}$ and $V_{\text{min}} = 0.242 \text{ mV}$. $V_{\text{max}} = 10.0043 \text{ V}$ is close to the value of $V_{\text{input}} = 10.0114 \text{ V}$; the discrepancy between the two is 0.0071 (0.07% error). The minimum resistance is not zero because some minimal resistance in the pot can not be avoided.

8. Adjust the shaft position of the pot to yield $V_o = 5 \text{ V}$. Then place a 10 kΩ resistor in parallel with $V_o$. What is the new value of $V_o$? Explain why there is this change. Compare the reading with the theoretically calculated $V_o$.

The shaft position is adjusted to read as close to 5 V as possible: $V_o = 5.0096 \text{ V}$. After we place a 10 kΩ resistor in parallel with $V_o$, the new value is: $V'_o = 3.9590 \text{ V}$. This change happens because the resistor $R_2$ is now replaced with $R_x$ (equivalent resistance of $R_2$ and $R_p$ in parallel).

$$R_x = R_2||R_p = \frac{R_2 \cdot R_p}{R_2 + R_p} = \frac{50 \text{ kΩ}}{10 \text{ kΩ} + 5 \text{ kΩ}} = 3.33 \text{ kΩ}$$
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\[ V_0 = V_i \left( \frac{R_x}{R_1 + R_x} \right) = 10.0114 \times \frac{3.33 \, k\Omega}{3.33 \, k\Omega + 5 \, k\Omega} = 4.006 \, V \]

The difference between this theoretical value and the measured value \( V_0 = 3.9590 \, V \) (1.1% error) is due to resistor value tolerance in the pot and human error in manipulation.

**B. Current Division**

1. Use the 0 to 6 volt Power Supply output and construct the voltage divider circuit as shown in Fig. 7. Turn the Power Supply voltage output to 0 volts while constructing the circuit. We will be simulating a constant current source by using a voltage source and adjusting the voltage as needed to create a predetermined current.

   ![Figure #7. A Current Divider](image)

2. Use \( R_1 = R_2 = 100 \, \Omega \).

   The resistors we use have 5% tolerance level (gold band).

3. Watching the display of the Power Supply, adjust the current \( I_s \) until it reaches 50mA.

   We adjusted the current to 50.139 mA DC. This gives us 0.139 mA error in measurements (0.3%) we should consider when explaining discrepancy later.
4. With the use of the DC ammeter verify KCL by measuring the current \( I_{R2} \) through the resistor \( R2 \).

Kirchhoff’s Current Law (KCL) states that the sum of all currents for any node is zero.

\[
I_s = 50.139 \text{ mA} \\
I_{R2} = -26.296 \text{ mA} \\
I_{R2} \times 2 = -52.592 \text{ mA} \\
\text{Sum} = -2.453 \text{ mA}
\]

This is close to zero, within 5% tolerance. Hence, KCL is satisfied. The discrepancy is due to the resistor tolerance and possible resistance in wires or bad connections.

5. Using the principle of current division calculate the theoretical value of \( I_{R2} \) and compare it with the measured value. Account for the causes of possible disagreements.

\[
I_{R2} = I_s \frac{R_2}{R_1 + R_2} = 50.139 \text{ mA} \frac{100 \Omega}{100 \Omega + 100 \Omega} = 25.0695 \text{ mA}
\]

The measured value is \( I_{R2} = 26.296 \text{ mA} \). The discrepancy is 1.226 mA (4.5% error), within 5% tolerance. This disagreement is possibly due to inaccuracy in measurement apparatus and resistor tolerances.

6. Leave \( R2 \) in place, repeat steps 4 and 5 for the case \( R1 = 10 \) \( \Omega \) and for the case \( R1 = 1 \text{ k}\) \( \Omega \). Be sure to maintain a constant current of 50 mA as viewed on the Power Supply display.

**CASE 1: \( R1=10\Omega, R2=100\Omega \)**

\[
I_s = -50.312 \text{ mA} \\
I_{R2} = 6.88 \text{ mA} \\
I_{R1} = 47.232 \text{ mA} \\
\text{Sum} = -2.453 \text{ mA}
\]

This is close to zero, within tolerance. Hence, KCL is satisfied. The discrepancy is due to the inaccuracy of the measuring equipment and resistance tolerance.

\[
I_{R2} = I_s \frac{R_1}{R_1 + R_2} = 50.312 \text{ mA} \frac{10 \Omega}{10 \Omega + 100 \Omega} = 4.573 \text{ mA}
\]
There is a discrepancy of 2.31 mA (50% error). This error is too high to be explained simply by the resistor tolerances or resistance in wires. (For example, even if we take that the actual resistor value is 95 Ω instead of 100 Ω, and plug-in exactly 50mA in the formula, the error goes down to 40%)
There is a possibility we made an error in this measurement.

CASE 2: R1=1kΩ, R2=100Ω

\[ I_s = -50.151 \text{ mA} \]
\[ I_{R2} = 45.66 \text{ mA} \]
\[ I_{R1} = 4.821 \text{ mA} \]
\[ \text{Sum} = 0.33 \text{ mA} \]
This is close to zero, within 5% tolerance. Hence, KCL is satisfied. The discrepancy is due to some resistance in the wires and resistor tolerance.

The measured value is \( I_{R2} = 45.66 \text{ mA} \). The discrepancy is 0.07 mA, within 5% tolerance. This disagreement is possibly due to resistance in wires and resistor tolerances.

C. Bridge Circuit

1. Using the principle of voltage division show that for the bridge circuit shown in Figure # 8, \( R_x = \frac{R_2 R_3}{R_1} \), if the voltage between points p and q is zero. When points p and q are at the same potential, the bridge circuit is balanced and the meter (either voltmeter or ammeter) will read 0.

We can see that the bridge circuit consists of two voltage dividers (R1 and R3, and R2 and Rx), driving the common voltage between points p and q. Applying the voltage divider formula twice we get:

\[ V_{pq} = \frac{R_2}{R_1+R_2} V \]

Where V is the input voltage. The bridge is said to be balanced when \( V_{pq} = 0 \), indicating that there is no difference in potential between points p and q.
2. Construct the bridge circuit with $R_1 = 2.2 \, k\Omega$, $R_2 = 4.7 \, k\Omega$, a decade resistor box for $R_3$ and the potentiometer for $R_x$. Set the decade box to about $3 \, k\Omega$ and the pot shaft position about 1/3 way of its full range.

3. Turn the power supply on and adjust the output to +10 V.

4. Monitor the voltage between points $p$ and $q$ with a DC voltmeter. Adjust the decade box resistance value until the voltmeter reading is as close to 0 as possible. Based on the formula given in step 1, find $R_x$.

We adjust the value of the decade box to 4,601 $\Omega$ instead of 3,000 $\Omega$. Applying the formula, we have:

$$R_x = \frac{R_2 R_3}{R_1} = \frac{4.7 \, k\Omega \times 4.6 \, k\Omega}{2.2 \, k\Omega} = 9.82 \, k\Omega.$$
5. Carefully remove the pot from the circuit without changing its shaft position. Now measure the resistance Rx using the ohmmeter and compare with the value obtained in the previous step. Discuss the causes of possible discrepancies. A bridge circuit can be used to make precision measurements of unknown resistance.

The measured resistance is Rx = 7.1058 kΩ. This represents a discrepancy of 2.714 kΩ from the previous measurement. The discrepancy in the measured value comes, in my opinion, mostly from human error contribution in handling the sensitive potentiometer. There is a possibility we have made a mistake in our measurement.

Conclusion

In this lab experiment we explored the concepts of voltage division and current division. We also tried to verify experimentally the two Kirchhoff’s laws: KVL and KCL.

In Part A we built a voltage divider circuit and using different resistor values each time, we confirmed KVL. We also tried the voltage divider formula both on a fixed voltage divider and on a variable voltage divider; all our measurements agreed with the theoretically calculated values, within 5% tolerance.

In Part B, using a current divider circuit and connecting different resistors, we verified KCL and the current divider formulas.

In Part C we built a bridge circuit and verified the condition for having a balanced bridge.

Reference


Attachments: Lab Notes
Experiment #3: Kirchhoff’s Laws

Report by: Name
Partner: Name

Tuesday, 2/14
* Intro to Spire
* Lab #3

SAMPLE

\[ V_{0} = \frac{R_{2}}{R_{1}} \times 9.9099 \text{ V} \]

\[ V_{0} = 0.00901 \text{ V} \]
D. Current Division

Adjusted to 50.120 mA ABC

C. Bridge Circuit

\[ R_x = \frac{R_2 R_3}{R_1} = 9.827 \text{ k\Omega} \]

Measured \[ R_x = 7.1058 \text{ k\Omega} \]
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Kirchhoff's Laws

SAMPLE

\[ R_x = \frac{10}{3} \, \text{K} \Omega \]

\[ V_0 = 10\, V, \quad R_x = \frac{10 \times 10}{25 + 5} = 4.00 \, \text{K} \Omega \]

\[ \text{Measured} = 5.95 \, \text{V} \]