Why do We Fight:
A Game Theoretic Analysis

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The Problem

Fighting between humans, human groups, or institutions
often involves ownership of an asset or the setting of a norm

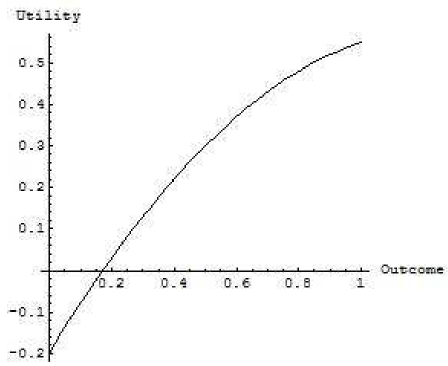
In many such situations the asset is divisible
and the norm is adjustable

But fighting is usually costly and hazardous
and its outcome could often be reached without fighting

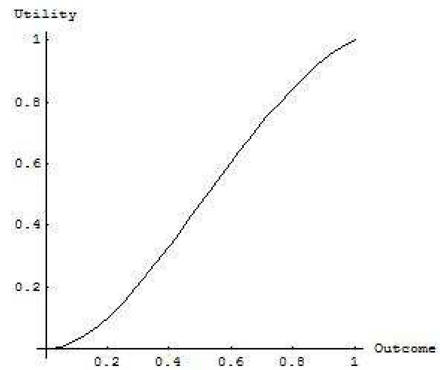
So, can fighting be *rational*? meaning:
can it result from the "intelligent" pursuit by each side of its best interests?

Game theory assumes rational decision makers (players) who seek to
maximize their utility (payoff) of the outcome (of the game)
Utility is a (real) valued function that increases with the players satisfaction

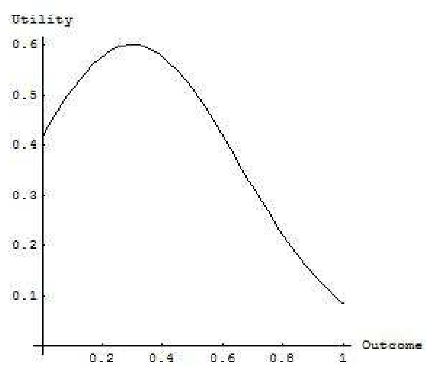
Examples of Utility Functions



Concave



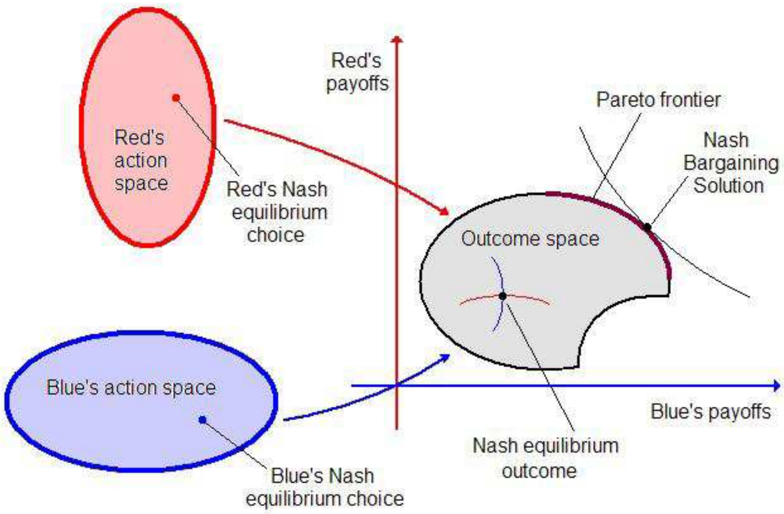
S-shaped



Quasi-concave

Game Theory in a Nutshell

Nash (1950)



A Nash equilibrium cannot be improved upon unilaterally

No joint improvement is possible on the Pareto frontier

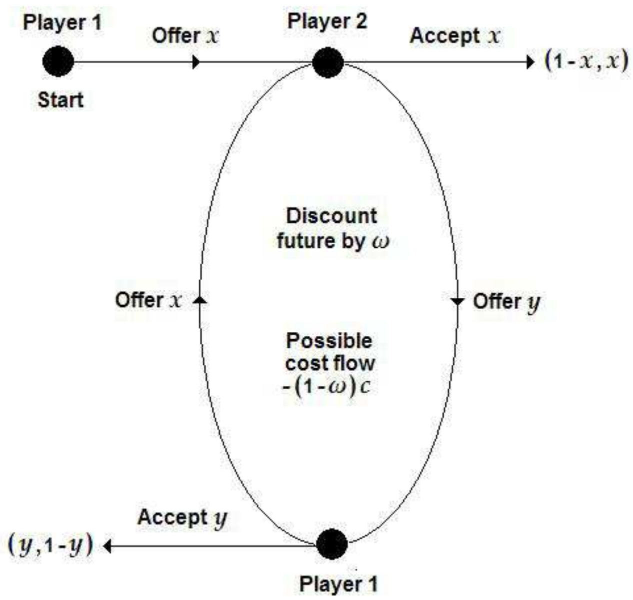
The Nash bargaining solution is a "fair" division of the pie

The "Nash program":

Explaining cooperative outcomes by egotistic choices

Strategic Bargaining

Rubinstein (1982):



The future is discounted at rate $\omega = e^{-r\Delta t} \in (0,1)$ per turn

An accepted offer ends the game

A *subgame perfect equilibrium* (Selten, 1975) is a Nash equilibrium

in "expected" utilities "at every turn"

If players value outcome z through a (weakly) concave utility $u_i(z)$

there is a unique SPE where Player 1 offers x such that

$$u_2(x) + (1-\omega)c = \omega u_2\Big(1 - u_1^{-1}\big(\omega u_1(1-x) \; - (1-\omega)c\big)\Big)$$

that Player 2 immediately accepts (it's efficient)

x converges to the Nash bargaining solution as $\Delta t \to 0$

Some Rationalist Approaches to Conflict

A "naive" approach:

A status quo is unacceptable to one side

The "challenger" makes a demand to the "defender"

They should negotiate (according to Rubinstein) since fighting is costly

So why should bargaining fail and fighting occur?

- There are indivisibilies (c.f. Salomon's Judgment)
- There are uncertainties on capabilities or resolve
- There are commitment problems (leading to salami tactics)

Other less naive approaches:

- The defender does not accept to negotiate (doorbell problem)

- The players do not agree to the Rubinstein (bargaining) framework

More realistic "bargaining" models might explain fighting

Some Issues with the Original Rubinstein Model

• Fixed unit period: you speak only at regular intervals of time

• Strictly alternating turns

• It is about sharing a surplus: no one is asked to *give up* something

• There is no uncertainty about the other side's parameters (u_i, ω_i, c_i)

Some desirable features:

• (Fixed) costs c_i replaced by utilities $U_i(y_i, y_j)$
of playing a (flow) game (with decisions y_i, y_j) through time
(Busch and Wen, 1995)

• Players can speak and move (on y_i, y_j) at any time t (continuous)
(Cramton, 1992, Smith & Stachetti, 2003)

• There is a status quo and one side is asked to surrender part of it
(Langlois & Langlois, 2005)

• There is uncertainty about the other side's parameters
(Harsanyi, 1965, and many followers)

Highly desirable feature:

A *single* model that has *all* the desirable features

A General Model (I)

The game unfolds within the time continuum $t \in [0, \infty)$

Each side i controls state variables $Y_i = Y_i(t)$ (level of fighting, etc.),

Each side values any current state $Y = (Y_i, Y_j) \in \mathcal{A}$ with flow utility $U_i(Y)$

Each side can make offers $X_i \in \mathcal{A}$ and can (finally) accept the other's offer

Offer and its acceptance can be simultaneous

There is no arbitration of incompatible moves

Explosive strategies are not allowed

Any change in $X_i,\ X_j,$ or Y is an "event"

At each time t there is a history h^t of "prior events"

The "strategies" induced by h^t

result in an expected future evolution σ^t (path) of the game

$$\sigma^t : \tau \rightarrow (Y^t(\tau), X_i^t(\tau), X_j^t(\tau))$$

deterministic or random in choice and/or timing

A leg is the time interval $(t, t+s)$ separating successive events

The path σ^t is thus made up of expected successive legs

A General Model (II)

In an SPE each side i at any time t is maximizing the "expected utility":

$$E_i^t = \int_0^\infty r_i e^{-r_i \tau} U_i(Y^t(\tau)) d\tau$$

where r_i is a measure of i's "impatience"

If an offer $X_j \in \mathcal{A}$ by side j is "expected" to be accepted by i at time $(t+s)$

$$E_i^t = \int_0^s r_i e^{-r_i \tau} U_i(Y^t(\tau)) d\tau + e^{-r_i s} U_i(X_j)$$

If the timing of that acceptance is given by a distribution function $F_i^t(s)$

and that of future offer $X_i^t(s)$ by j is given by $F_j^t(s)$ the expected utility is

$$E_i^t = \int_{[0,\infty)} e^{-r_i s} dG_{ij}^t(s) \qquad \text{where}$$

$$dG_{ij}^t(s) = U_i\big(X_j^t(s)\big) dF_i^t(s) + U_i\big(X_i^t(s)\big) dF_j^t(s) + r_i U_i\big(Y^t(s)\big)\Phi^t(s) ds$$

where $\Phi^t(s) = 1 - F_i^t(s) - F_j^t(s)$

and $X_i^t(s)$, $X_j^t(s)$ are the expected offers

E_i^t is a Lebesgues-Stieltjes (Radon) integral

Some Methodology

If $(t, t+\theta)$ is the first leg of the path then (dynamic programming)

$$E_i^t = \Delta G_{ij}^t(0) + \int_{(0,\theta)} e^{-r_i s} dG_{ij}^t(s) + e^{-r_i \theta}\Phi^t(\theta)E_i^{t+\theta}$$

$\Delta G_{ij}^t(0)$ is a "mass" resulting from discontinuities $\Delta F_i^t(0)$ and/or $\Delta F_j^t(0)$

Within the leg $(t, t+\theta)$ offers $X_i^t,\ X_j^t$ and state variable Y^t are constant and probabilities of acceptance are continuous

Let $\phi_i^t(s)$ be i's "survival" function (probability i does not accept) on the leg

Then $dF_j^t(s) = -\Phi^t(0^+)\phi_i^t(s)d\phi_j^t(s)$ for $s \in (0,\theta)$

On any leg $(t, t+\theta)$ in an SPE either

- The two sides are non-acceptant: $\phi_i^t(s) \equiv 1$; or
- The two sides "countervail" each other: $E_i^{t+s} = U_i(X_j^t)$

Some Results

On a *deterministic* (θ not random) leg, countervailing requires

$$\phi_j^t(s) = e^{-\lambda_i s} \qquad \text{with} \qquad \lambda_i = r_i \frac{U_i(X_j^t) - U_i(Y^t)}{U_i(X_i^t) - U_i(X_j^t)}$$

If a new offer $X_j^{t+\theta}$ can be made at a random time θ

of exponential distribution of parameter ρ_j, countervailing requires

$$\lambda_i = \frac{r_i\big(U_i(X_j^t) - U_i(Y^t)\big) - \rho_j\big(U_i(X_j^{t+\theta}) - U_i(X_j^t)\big)}{U_i(X_i^t) - U_i(X_j^t)}$$

If offers satisfy

$U_i(X_i^t) > U_i(X_j^t)$ (reasonable) and

$r_i\big(U_i(X_j^t) - U_i(Y^t)\big) \geq \rho_j\big(U_i(X_j^{t+\theta}) - U_i(X_j^t)\big)$

on each leg, **then countervailing provides a SPE**

Non-acceptant behavior cannot yield a SPE in the long-run

because it requires $E_i^{t+s} > U_i(X_j^t)$ and therefore better and better offers

and countervailing must eventually tick in

So, bargaining failure (disagreement on offers) and fighting

(if Y^t amounts to fighting) is a perfectly rational behavior

Strict timing assumptions in Rubinstein yields artificial results

The Issue of Information

(Fighting *can* be rational but why should it *prevail*?)

There are now two (finite) sets of types, with $i \in \mathcal{I}$ and $j \in \mathcal{J}$

At time t there are beliefs $\left(b_j(t)\right)_{j\in\mathcal{J}}$ ($b_j(t) \geq 0, \sum_{j\in\mathcal{J}} b_j(t) = 1$) about side $\mathcal{J}$

Assume common *Nash equilibrium* Y and extreme offers $\Xi_{\mathcal{I}},\ \Xi_{\mathcal{J}}$

Continuous-time Bayesian updating (focusing on acceptance) reads

$$b_j(t+s) = \frac{b_j(t)\phi_j^t(s)}{\phi_{\mathcal{J}}^t(s)} \qquad \text{with} \quad \phi_{\mathcal{J}}^t(s) = \sum_{k\in\mathcal{J}} b_k(t)\phi_k^t(s)$$

Dynamic programming still holds (with some continuity assumption):

$$E_i^t = \sum_{j\in\mathcal{J}} b_j(t) \int_{[0,\theta)} e^{-r_i s} dG_{ij}^t(s) + e^{-r_i\theta}\phi_i^t(\theta)\phi_{\mathcal{J}}^t(\theta)E_i^{t+\theta}$$

Side $\mathcal{J}$ offers $\Xi_{\mathcal{J}}$ and uses its Nash equilibrium choice $Y_{\mathcal{J}}$

For each type $i \in \mathcal{I}$ now let $\lambda_i = r_i \frac{U_i(\Xi_{\mathcal{J}}) - U_i(Y)}{U_i(\Xi_{\mathcal{I}}) - U_i(\Xi_{\mathcal{J}})}$

An "active type" $i = \underset{l\,\in\,\mathcal{I}}{\text{argmax}}\{\lambda_l | b_l(t) > 0\}$ is identified on each side at each t

Current "active type" $j \in \mathcal{J}$ uses $\phi_j^t(s) = \frac{e^{-\lambda_i s} + b_j(t) - 1}{b_j(t)}$ so that $\phi_{\mathcal{J}}^t(s) = e^{-\lambda_i s}$

while others are non-acceptant (i.e., $\phi_k^t(s) \equiv 1$)

Together with Bayesian updating this forms a Perfect Bayesian equilibrium

Some Empirical Results

$$\phi_j^t(s) = e^{-\lambda_i s} \qquad \text{with} \qquad \lambda_i = r_i \frac{U_i(X_j^t) - U_i(Y^t)}{U_i(X_i^t) - U_i(X_j^t)}$$

is a testable relationship using survival analysis techniques

λ_i is called a "hazard rate" (in duration models)

It should be related to the utility structure of the game

We use a classic sanctions data base developed by Hufbauer, Schott and Elliott and enriched by Drezner

In economic sanctions a sanctioner (sender) is imposing costly sanctions on a target

Sanctions are often costly to the sender as well

The relationship should hold when both sides suffer and may fail if the sender does not suffer

Statistical Results

Estimated Hazard when Target Acquiesces to Sender Demand

	Costly Sanctions Cases	
Explanatory Variables	Estimated Coefficient	t-statistic
CSENDER	0.9995**	2.10
CTARGET	– 0.5254**	– 3.54
ALLY	4.3734**	3.38
COOPERATION	– 0.8049*	– 1.95
Constant	– 11.874**	– 3.28
Duration dependence parameter p	3.3967**	4.35
Number of cases	14	
log Likelihood	– 6.1637**	
*p $<$ 0.10, **p $<$ 0.05 are two tailed significance levels.		

Estimated Hazard when Sanctions End with

Sanctioner's acceptance of a Compromise or a Return to the *Status Quo*

	Costly Sanctions Cases	
Explanatory Variables	Estimated Coefficient	t-statistic
CTARGET	0.4537**	2.73
OUTCOME	– 0.2655**	– 2.00
ASSIST	1.4528**	2.83
Constant	– 1.6041**	– 3.40
Duration dependence parameter, p	1.1704**	7.27
Number of cases	36	
log Likelihood	– 50.5587**	
*p $<$ 0.10, **p $<$ 0.5 are significance levels on two tailed tests.		

Similar tests on the non-costly case yield non-significant results

A Darwinian Perspective

Economic sanctions are relatively "rare" events

But bargaining with fixed costs is an everyday experience for many

In the Rubinstein case re-framed in continuous time

$$\lambda_i = r_i \frac{c_i + u_i(x_j^t)}{u_i(1-x_i^t) - u_i(x_j^t)}$$

$r_i\big(c_i + u_i(x_j^t)\big)dt$ is the total cost of waiting an additional dt

$u_i(1-x_i^t) - u_i(x_j^t)$ reflects how far apart the two sides are

If x_i^t and x_j^t remain fixed until either side accepts at time $(t+\theta)$

the results for i viewed *at the time of acceptance* are

$$P_i(A_j^\theta) = u_i(1-x_i^t) - r_i c_i \int_0^\theta e^{r_i(\theta - s)} ds = u_i(1-x_i^t) - c_i(e^{r_i\theta} - 1)$$

if j accepts, and $P_i(A_i^\theta) = u_i(x_j^t) - c_i(e^{r_i\theta} - 1)$ if i accepts

The expected result for i viewed *at the time of acceptance* is therefore

$$EP_i = \int_0^\infty \big(\lambda_i P_i(A_j^\theta) + \lambda_j P_i(A_i^\theta)\big) e^{-(\lambda_i + \lambda_j)\theta} d\theta$$

$$= \frac{\lambda_i u_i(1-x_i^t) + \lambda_j u_i(x_j^t)}{\lambda_i + \lambda_j} + \frac{r_i c_i}{r_i - \lambda_i - \lambda_j}$$

The best survivors are those who get the highest $EP_i = EP_i(x_i^t, x_j^t)$

It turns out that EP_i is quasi-concave in x_i^t and the Nash equilibrium

of the "ex-post" game defined by EP_i involves ungenerous offers

Player Moves as Logical Instructions

How can an offer and its acceptance be simultaneous?

By extending the player's action space to logical instructions such as:

• I accept at least this much instantly

• Here is my offer, take it or leave it

So, choice, offer, and acceptance "moves"

are only a universe of discourse $\mathcal{U}$

A player's decision $\alpha_i^t \in \Omega_i$ by i at time t is

a set of logical instructions (a computer program)

And it can use the other side's instructions as input

$\alpha_i^t : \alpha_j^t \in \Omega_j \rightarrow$ (state choice, offer, and acceptance) $\in \mathcal{A}_i$

Example:

$\alpha_i^t:$ (import α_j^t)

if $(\alpha_j^t(i$ accepts$) \rightarrow j$ offers $X_j)$ and $(U_i(X_j) \geq z)$

then accept X_j else reject

$\alpha_j^t:$ (import α_i^t)

if $(\alpha_i^t(j$ offers $X_j) \rightarrow i$ accepts$)$

then offer X_j else make no offer