Why do We Fight:  
A Game Theoretic Analysis

by
Jean-Pierre Langlois
Mathematics Department
San Francisco State University
The Problem

Fighting between humans, human groups, or institutions often involves ownership of an asset or the setting of a norm.

In many such situations the asset is divisible and the norm is adjustable.

But fighting is usually costly and hazardous and its outcome could often be reached without fighting.

So, can fighting be rational? meaning: can it result from the "intelligent" pursuit by each side of its best interests?

Game theory assumes rational decision makers (players) who seek to maximize their utility (payoff) of the outcome (of the game). Utility is a (real) valued function that increases with the players satisfaction.
Examples of Utility Functions

Concave

S-shaped

Quasi-concave
A Nash equilibrium cannot be improved upon unilaterally
No joint improvement is possible on the Pareto frontier
The Nash bargaining solution is a "fair" division of the pie
The "Nash program"
Explaining cooperative outcomes by egotistic choices
Strategic Bargaining

Rubinstein (1982):

The future is discounted at rate $\omega = e^{-r\Delta t} \in (0, 1)$ per turn

An accepted offer ends the game

A subgame perfect equilibrium (Selten, 1975) is a Nash equilibrium
in "expected" utilities "at every turn"

If players value outcome $z$ through a (weakly) concave utility $u_i(z)$

there is a unique SPE where Player 1 offers $x$ such that

$$u_2(x) + (1 - \omega)c = \omega u_2\left(1 - u_1^{-1}(\omega u_1(1 - x) - (1 - \omega)c)\right)$$

that Player 2 immediately accepts (it's efficient)

$x$ converges to the Nash bargaining solution as $\Delta t \to 0$
Some Rationalist Approaches to Conflict

A "naive" approach:

A status quo is unacceptable to one side

The "challenger" makes a demand to the "defender"

They should negotiate (according to Rubinstein) since fighting is costly

So why should bargaining fail and fighting occur?

• There are indivisibilities (c.f. Salomon's Judgment)
• There are uncertainties on capabilities or resolve
• There are commitment problems (leading to salami tactics)

Other less naive approaches:

• The defender does not accept to negotiate (doorbell problem)
• The players do not agree to the Rubinstein (bargaining) framework

More realistic "bargaining" models might explain fighting
Some Issues with the Original Rubinstein Model

- Fixed unit period: you speak only at regular intervals of time
  - Strictly alternating turns
- It is about sharing a surplus: no one is asked to give up something
- There is no uncertainty about the other side's parameters ($u_i$, $\omega_i$, $c_i$)

Some desirable features:

- (Fixed) costs $c_i$ replaced by utilities $U_i(y_i, y_j)$ of playing a (flow) game (with decisions $y_i, y_j$) through time
  (Busch and Wen, 1995)
- Players can speak and move (on $y_i, y_j$) at any time $t$ (continuous)
  (Cramton, 1992, Smith & Stachetti, 2003)
- There is a status quo and one side is asked to surrender part of it
  (Langlois & Langlois, 2005)
- There is uncertainty about the other side's parameters
  (Harsanyi, 1965, and many followers)

Highly desirable feature:

A single model that has all the desirable features
A General Model (I)

The game unfolds within the time continuum $t \in [0, \infty)$

Each side $i$ controls state variables $Y_i = Y_i(t)$ (level of fighting, etc.),

Each side values any current state $Y = (Y_i, Y_j) \in \mathcal{A}$ with flow utility $U_i(Y)$

Each side can make offers $X_i \in \mathcal{A}$ and can (finally) accept the other's offer

Offer and its acceptance can be simultaneous

There is no arbitration of incompatible moves

Explosive strategies are not allowed

Any change in $X_i, X_j$, or $Y$ is an "event"

At each time $t$ there is a history $h^t$ of "prior events"

The "strategies" induced by $h^t$

result in an expected future evolution $\sigma^t$ (path) of the game

$\sigma^t : \tau \rightarrow (Y^t(\tau), X_i^t(\tau), X_j^t(\tau))$

deterministic or random in choice and/or timing

A leg is the time interval $(t, t + s)$ separating successive events

The path $\sigma^t$ is thus made up of expected successive legs
A General Model (II)

In an SPE each side $i$ at any time $t$ is maximizing the "expected utility":

$$E_i^t = \int_0^\infty r_i e^{-r_i \tau} U_i(Y^t(\tau)) d\tau$$

where $r_i$ is a measure of $i$’s "impatience"

If an offer $X_j \in \mathcal{A}$ by side $j$ is "expected" to be accepted by $i$ at time $(t + s)$

$$E_i^t = \int_0^s r_i e^{-r_i \tau} U_i(Y^t(\tau)) d\tau + e^{-r_i s} U_i(X_j)$$

If the timing of that acceptance is given by a distribution function $F_i^t(s)$ and that of future offer $X_i^t(s)$ by $j$ is given by $F_j^t(s)$ the expected utility is

$$E_i^t = \int_{[0,\infty)} e^{-r_i \tau} dG_{ij}^t(s)$$

where

$$dG_{ij}^t(s) = U_i(X_j^t(s)) dF_i^t(s) + U_i(X_i^t(s)) dF_j^t(s) + r_i U_i(Y^t(s)) \Phi^t(s) ds$$

where $\Phi^t(s) = 1 - F_i^t(s) - F_j^t(s)$

and $X_i^t(s), X_j^t(s)$ are the expected offers

$E_i^t$ is a Lebesgues-Stieltjes (Radon) integral
**Some Methodology**

If \((t, t + \theta)\) is the first leg of the path then (dynamic programming)

\[
E_i^t = \Delta G_{ij}^t(0) + \int_{(0, \theta)} e^{-r,s} dG_{ij}^t(s) + e^{-r,\theta} \Phi^t(\theta) E_i^{t+\theta}
\]

\(\Delta G_{ij}^t(0)\) is a "mass" resulting from discontinuities \(\Delta F_i^t(0)\) and/or \(\Delta F_j^t(0)\)

Within the leg \((t, t + \theta)\) offers \(X_i^t, X_j^t\) and state variable \(Y^t\) are constant and probabilities of acceptance are continuous

Let \(\phi_i^t(s)\) be \(i\)'s "survival" function (probability \(i\) does not accept) on the leg

Then \(dF_j^t(s) = -\Phi^t(0^+) \phi_i^t(s) d\phi_j^t(s)\) for \(s \in (0, \theta)\)

On any leg \((t, t + \theta)\) in an SPE either

- The two sides are non-acceptant: \(\phi_i^t(s) \equiv 1\); or
- The two sides "countervail" each other: \(E_i^{t+s} = U_i(X_j^t)\)
Some Results

On a deterministic (θ not random) leg, countervailing requires

\[ \phi_j^t(s) = e^{-\lambda_is} \quad \text{with} \quad \lambda_i = \frac{r_i(U_i(X_i^t) - U_i(Y^t))}{U_i(X_i^t) - U_i(X_j^t)} \]

If a new offer \( X_j^{t+\theta} \) can be made at a random time \( \theta \) of exponential distribution of parameter \( \rho_j \), countervailing requires

\[ \lambda_i = \frac{r_i(U_i(X_j^t) - U_i(Y^t)) - \rho_j(U_i(X_j^{t+\theta}) - U_i(X_j^t))}{U_i(X_i^t) - U_i(X_j^t)} \]

If offers satisfy

\[ U_i(X_i^t) > U_i(X_j^t) \quad \text{(reasonable)} \quad \text{and} \quad r_i(U_i(X_j^t) - U_i(Y^t)) \geq \rho_j(U_i(X_j^{t+\theta}) - U_i(X_j^t)) \]

on each leg, then countervailing provides a SPE

Non-acceptant behavior cannot yield a SPE in the long-run because it requires \( E_i^{t+s} > U_i(X_j^t) \) and therefore better and better offers and countervailing must eventually tick in

So, bargaining failure (disagreement on offers) and fighting (if \( Y^t \) amounts to fighting) is a perfectly rational behavior

Strict timing assumptions in Rubinstein yields artificial results
The Issue of Information

(Fighting can be rational but why should it prevail?)

There are now two (finite) sets of types, with \( i \in \mathcal{I} \) and \( j \in \mathcal{J} \)
At time \( t \) there are beliefs \( \left( \frac{b_j(t)}{j \in \mathcal{J}} \right) \) \( b_j(t) \geq 0, \sum_{j \in \mathcal{J}} b_j(t) = 1 \) about side \( \mathcal{J} \)

Assume common Nash equilibrium \( Y \) and extreme offers \( \Xi_{\mathcal{I}}, \Xi_{\mathcal{J}} \)
Continuous-time Bayesian updating (focusing on acceptance) reads

\[
 b_j(t + s) = \frac{b_j(t) \phi_j^t(s)}{\phi_j^t(s)} \quad \text{with} \quad \phi_j^t(s) = \sum_{k \in \mathcal{J}} b_k(t) \phi_k^t(s)
\]

Dynamic programming still holds (with some continuity assumption):

\[
 E_i^t = \sum_{j \in \mathcal{J}} b_j(t) \int_{[0,\theta]} e^{-r_i \lambda} dG_i^t(s) + e^{-r_i \theta} \phi_i^t(\theta) \phi_j^t(\theta) E_i^{t+\theta}
\]

Side \( \mathcal{J} \) offers \( \Xi_{\mathcal{J}} \) and uses its Nash equilibrium choice \( Y_{\mathcal{J}} \)

For each type \( i \in \mathcal{I} \) now let \( \lambda_i = \frac{U_i(\Xi_{\mathcal{J}}) - U_i(Y)}{U_i(\Xi_{\mathcal{I}}) - U_i(\Xi_{\mathcal{J}})} \)

An "active type" \( i = \arg\max_{l \in \mathcal{I}} \{ \lambda_l | b_l(t) > 0 \} \) is identified on each side at each \( t \)

Current "active type" \( j \in \mathcal{J} \) uses \( \phi_j^t(s) = \frac{e^{-\lambda_i s} + b_j(t) - 1}{b_j(t)} \) so that \( \phi_j^t(s) = e^{-\lambda_i s} \)

while others are non-acceptant (i.e., \( \phi_k^t(s) \equiv 1 \))

Together with Bayesian updating this forms a Perfect Bayesian equilibrium
Some Empirical Results

\[ \phi^t_j(s) = e^{-\lambda_i s} \quad \text{with} \quad \lambda_i = r_i \frac{U_i(X'_i) - U_i(Y'_i)}{U_i(X'_i) - U_i(X'_j)} \]

is a testable relationship using survival analysis techniques.

\( \lambda_i \) is called a "hazard rate" (in duration models).

It should be related to the utility structure of the game.

We use a classic sanctions data base developed by Hufbauer, Schott and Elliott and enriched by Drezner.

In economic sanctions a sanctioner (sender) is imposing costly sanctions on a target.

Sanctions are often costly to the sender as well.

The relationship should hold when both sides suffer

and may fail if the sender does not suffer.
### Statistical Results

Estimated Hazard when Target Acquiesces to Sender Demand

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Estimated Coefficient</th>
<th>t-statistic</th>
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<tbody>
<tr>
<td>CSENDER</td>
<td>0.9995**</td>
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<td>CTARGET</td>
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<td>COOPERATION</td>
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<tr>
<td>log Likelihood</td>
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*p < 0.10, **p < 0.05 are two tailed significance levels.

Estimated Hazard when Sanctions End with Sanctioner's acceptance of a Compromise or a Return to the Status Quo

<table>
<thead>
<tr>
<th>Explanatory Variables</th>
<th>Estimated Coefficient</th>
<th>t-statistic</th>
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<td>CTARGET</td>
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<td>log Likelihood</td>
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*p < 0.10, **p < 0.5 are significance levels on two tailed tests.

Similar tests on the non-costly case yield non-significant results.
A Darwinian Perspective

Economic sanctions are relatively "rare" events
But bargaining with fixed costs is an everyday experience for many

In the Rubinstein case re-framed in continuous time
\[ \lambda_i = r_i \frac{c_i + u_i(x_j^t)}{u_i(1-x_i^t) - u_i(x_j^t)} \]

\[ r_i (c_i + u_i(x_j^t)) dt \] is the total cost of waiting an additional \( dt \)
\[ u_i(1 - x_i^t) - u_i(x_j^t) \] reflects how far apart the two sides are

If \( x_i^t \) and \( x_j^t \) remain fixed until either side accepts at time \( (t + \theta) \)
the results for \( i \) viewed at the time of acceptance are
\[ P_i(A_j^\theta) = u_i(1 - x_i^t) - r_i c_i \int_0^\theta e^{r_i(\theta - s)} ds = u_i(1 - x_i^t) - c_i (e^{r_i \theta} - 1) \]
if \( j \) accepts, and \[ P_i(A_i^\theta) = u_i(x_j^t) - c_i (e^{r_i \theta} - 1) \] if \( i \) accepts

The expected result for \( i \) viewed at the time of acceptance is therefore
\[ EP_i = \int_0^\infty (\lambda_i P_i(A_j^\theta) + \lambda_j P_i(A_i^\theta)) e^{-(\lambda_i + \lambda_j)\theta} d\theta = \frac{\lambda_i u_i(1-x_i^t) + \lambda_j u_i(x_j^t)}{\lambda_i + \lambda_j} + \frac{r_i c_i}{r_i - \lambda_i - \lambda_j} \]

The best survivors are those who get the highest \( EP_i = EP_i(x_i^t, x_j^t) \)
It turns out that \( EP_i \) is quasi-concave in \( x_i^t \) and the Nash equilibrium of the "ex-post" game defined by \( EP_i \) involves ungenerous offers
Player Moves as Logical Instructions

How can an offer and its acceptance be simultaneous? By extending the player's action space to logical instructions such as:

- I accept at least this much instantly
- Here is my offer, take it or leave it

So, choice, offer, and acceptance "moves" are only a universe of discourse $\mathcal{U}$

A player's decision $\alpha^t_i \in \Omega_i$ by $i$ at time $t$ is a set of logical instructions (a computer program)

And it can use the other side's instructions as input $\alpha^t_i : \alpha^t_j \in \Omega_j \rightarrow (\text{state choice, offer, and acceptance}) \in \mathcal{A}_i$

Example:

$\alpha^t_i : (\text{import } \alpha^t_j)$
if $(\alpha^t_j(i \text{ accepts}) \rightarrow j \text{ offers } X_j)$ and $(U_i(X_j) \geq z)$ then accept $X_j$ else reject

$\alpha^t_j : (\text{import } \alpha^t_i)$
if $(\alpha^t_i(j \text{ offers } X_j) \rightarrow i \text{ accepts})$ then offer $X_j$ else make no offer