1. Evaluate \( \int \frac{\sqrt{\arctan x}}{1+x^2} \, dx \)

Use substitution \( u = \arctan(x), \, du = dx/(1 + x^2). \) So
\[
\int \frac{\sqrt{\arctan x}}{1+x^2} \, dx = \int \sqrt{u} \, du = \frac{1}{3} u^{3/2} + c = \frac{1}{3} (\arctan x)^{3/2} + c
\]

2. Evaluate \( \int (\ln x)^2 \, dx \)

By parts: \( u = \ln x, \, dv = \frac{1}{x} \, dx, \) so \( du = \frac{1}{x} \, dx \) and \( v = x(\ln x - 1) \) (see note below). Therefore
\[
\int (\ln x)^2 \, dx = x(\ln x)(\ln x - 1) - \int (\ln x - 1) \, dx
\]
\[
= x(\ln x)(\ln x - 1) - x(\ln x - 1) + x + c
\]
\[
= x((\ln x)^2 - 2\ln x + 2) + c
\]

Note: \( \int 1.\ln x \, dx = x\ln x - \int \frac{x}{x} \, dx = x(\ln x - 1) \) by parts.

3. Evaluate \( \int \sec^4 \theta \, d\theta \)

\[
\int \sec^4 \theta \, d\theta = \int (\sec^2 \theta)(\sec^2 \theta) \, d\theta = \int (1 + \tan^2 \theta)(\sec^2 \theta) \, d\theta
\]
\[
= \int (1 + u^2) \, du = u + \frac{1}{3} u^3 + c = \tan \theta + \frac{1}{3} \tan^3 \theta + c
\]
with \( u = \tan \theta. \)

4. Evaluate \( \int \frac{x^2}{(1+x)^3} \, dx \)

By partial fractions
\[
\frac{x^2}{(1+x)^3} = \frac{A}{(1+x)} + \frac{B}{(1+x)^2} + \frac{C}{(1+x)} = \frac{(A+B+C)+(B+2C)x+Cx^2}{(1+x)^3}
\]
So, \( A = 1, B = -2, C = 1, \) and
\[
\int \frac{x^2}{(1+x)^3} \, dx = \int \frac{1}{(1+x)} \, dx - 2 \int \frac{1}{(1+x)^2} \, dx + \int \frac{1}{(1+x)} \, dx
\]
\[
= - \frac{1}{2(1+x)^2} + \frac{2}{1+x} + \ln |1+x| + c
\]

5. Evaluate \( \int_0^1 \frac{\ln x}{\sqrt{x}} \, dx \)

By parts \( \int_0^1 \frac{\ln x}{\sqrt{x}} \, dx = 2x^{1/2} \ln x - 2 \int_0^1 \frac{x^{1/2}}{x} \, dx = 2x^{1/2} \ln x - 4x^{1/2} + c \)
with $u = \ln x$, $dv = x^{-1/2}$ so that $du = \frac{dx}{x}$ and $v = 2x^{1/2}$. So
\[
\int_0^1 \frac{\ln x}{\sqrt{x}} \, dx = \left(2x^{1/2}\ln x - 4x^{1/2}\right)|_0^1 = -4 - 2 \lim_{x \to 0^+} x^{1/2}\ln x = -4
\]
since $\lim_{x \to 0^+} x^{1/2}\ln x = \lim_{x \to 0^+} \frac{\ln x}{x^{1/2}} = \lim_{x \to 0^+} \frac{1/x}{x^{3/2} - (1/2)x^{3/2}} = -2 \lim_{x \to 0^+} x^{1/2} = 0$
by l'Hopital. So, the improper integral is convergent and has value $-4$.

6. Sketch the region $R$ bounded by $x = 0$, $x = 1$, $y = 0$, $y = e^{-x}$ and find the volume obtained by revolving $R$ about the $y$-axis.

By the shell method, the volume is
\[
V = 2\pi \int_0^1 xe^{-x} \, dx = -2\pi (x + 1)e^{-x}|_0^1 = 2\pi \left(1 - \frac{2}{e}\right)
\]
where $\int xe^{-x} \, dx = -xe^{-x} + \int e^{-x} \, dx = -(x + 1)e^{-x}$ by parts.