Game Theory (continued)

There are two main sources to the so-called evolutionary approach in game theory. On the one hand, Robert Axelrod, a political scientist, organized a tournament (in the late 1970's) where academics were invited to submit strategies for a repeated prisoner's dilemma tournament. 13 strategies were submitted in the first tournament with Tit-for-Tat (submitted by Anatol Rappoport) winning the field. Later, an expanded tournament was held with very similar results. Since then, numerous tournaments have been held with various changes in parameters. TFT usually does very well, but so do other strategies (such as Tit-for-two-Tat) depending on the circumstances.

On the other hand, John Maynard Smith, a population biologist, investigated socially stable strategies (also in the 1970's) and developed the concept of evolutionary stable strategy that I will discuss below.

More recently, the field of *Evolutionary Game Theory* has developed with contributions from a diverse group of researchers ranging from economics, to political science, biology, and mathematics.

The basic idea underlying most of evolutionary game theory is that a game will be played by a population of players over time. In most cases, the game is symmetric and players are paired randomly. As play proceeds and payoffs accumulate, some strategies do better than others. A selection mechanism then weeds out the poor performers and favors the good ones. The main concept underlying the theory was put forward by Maynard-Smith:

A strategy X is called an *evolutionary stable strategy* (ESS) if for any other strategy Y and any $\epsilon > 0$, X does better than Y against a population mix $(1-\epsilon)\mathrm{X} + \epsilon\mathrm{Y}$. Formally:

$$\mathrm{W}_i[\mathrm{X},(1-\epsilon)\mathrm{X} + \epsilon\mathrm{Y}] > \mathrm{W}_i[\mathrm{Y},(1-\epsilon)\mathrm{X} + \epsilon\mathrm{Y}]$$

where W_i represents the average payoff to the user of X or Y against the population mix $(1-\epsilon)\mathrm{X} + \epsilon\mathrm{Y}$. This may be the one-shot payoff or the limit average payoff mentioned before. To understand better this concept, one can investigate the effect of letting $\epsilon \to 0$:

$$\mathrm{W}_i[\mathrm{X},\mathrm{X}] \geq \mathrm{W}_i[\mathrm{Y},\mathrm{X}]$$

This means (since Y is arbitrary) that X is a Nash equilibrium against itself in the symmetric game. So, an ESS is a particular case (a refinement of) the Nash equilibrium. Let us investigate some examples:

Example 1: Suppose that our favorite prisoner's dilemma is played by "stubborn" players who may use pure or mixed strategies. We already know that $(Dfct, Dfct)$ is the sole Nash equilibrium of the one shot game. But since $Dfct$ is a dominant strategy:

$$W_i[X,X] > W_i[Y,X] \quad \text{and} \quad W_i[X,Y] > W_i[Y,Y]$$

and $W_i[X,(1-\epsilon)X+\epsilon Y] > W_i[Y,(1-\epsilon)X+\epsilon Y]$ for any ϵ. So, $Dfct$ appears to be ESS in this case.

Example 2: In the Rock-Scissor-Paper game, you must have found the sole Nash equilibrium $X = (\frac{1}{3},\frac{1}{3},\frac{1}{3})$. However, *any* other choice Y brings the *same* payoff 0. So, the equality $W_i[X,X] = W_i[Y,X]$ always holds. The only way to ensure the define requirement for an ESS is then $W_i[X,Y] > W_i[Y,Y]$. But here again, this fails since

$$W_i[X,Y] = W_i[Y,Y] = 0$$

for *any* Y. So, this game *does not* have an ESS.

Example 3: Let us return to the prisoner's dilemma but let us relax the stubbornness assumption. If one player chooses $Dfct$ all the time we will call that strategy ALLD. How would our favorite TFT fare against it? One easily finds that

$$W_i[TFT,ALLD] = -1$$
$$W_i[TFT,TFT] = 0$$
$$W_i[ALLD,TFT] = -1$$
$$W_i[ALLD,ALLD] = -1$$

So

$$W_i[TFT,(1-\epsilon)TFT+\epsilon ALLD] = (1-\epsilon)0+\epsilon(-1) >$$
$$W_i[ALLD,(1-\epsilon)TFT+\epsilon ALLD] = -1$$

for any $\epsilon > 0$. In other words, a population of ALLD players is easily invaded by a population of TFT players, even if it starts with very few members.

So, is TFT an ESS? Not really since one easily verifies that other strategies (such as ALLC) do just as well against it. But generally speaking, strategies that have properties similar to TFT do extremely well in prisoner's dilemma tournaments.

The question, however, becomes more complicated when a little bit of noise is introduced into the problem. Even if a player intends to cooperate, it is not always perceived as such by others. How does that affect the above analysis?