Chapter Six: Strategic Bargaining*

Bargaining is one of the most challenging topics of game theory. The raw problem can be described as follows: two individuals must decide how to divide a prize between themselves. If they can't agree on a division neither of them gets anything. So, both sides face a tension between getting more for themselves, at the expense of the other, but risking to get nothing at all if they can't agree.

The earliest attempt to model bargaining is due to John Nash who constructed a non-strategic, axiomatic-based approach in 1950. It involved so-called "Nash products" that continue to be used in models where bargaining is not the main focus but where some bargained outcomes need to be involved. The first truly strategic approach to bargaining is due to Rubinstein, in 1982.[1] It involves a discounted repeated game with the two bargainers alternating their offers until one accepts the latest one, thereby ending the game.

Bargaining can be part of a more complex situation. One of the main examples can be found in the literature on Bargaining and War: Two nation-states are involved in a dispute that can lead them to fight over a prize. They may also negotiate a peaceful agreement that would avoid the costs and vagaries of war. In its ultimate version, bargaining and war also involves uncertainties over the capabilities and priorities of the protagonists.

The Rubinstein strategic bargaining model fits quite well the problems typically encountered in Economics, when a prize not yet enjoyed by either side must be optimally divided. But in the case of an existing status quo, largely enjoyed by one side, the other dissatisfied side can threaten or engage in the use of force in order to obtain concessions and better share of the prize. As we show, later in this chapter, much of the Rubinstein logic can be extended to that case with the expected costs and vagaries of war accounted for in any settlement.

6.1 The Rubinstein Model

The two sides take turns making the offers $x \in [0,1]$ (by Blue to Red) and $y \in [0,1]$ (by Red to Blue). When an offer is accepted by one side, the other gets the complement of a prize of size arbitrarily set to 1. In the simplest case discussed here, the two sides discount the future by some common factor $\delta \in (0,1)$ and one's utility is identical to the size of one's share.[2] Since the game is repeated identically to itself until

[1]An earlier attempt by Ståhl using a finite game tree was far less convincing because the solution depended on the tree length set by the theorist.

[2]This is known as "risk-neutral" utilities. Note that enjoying x forever, with discount factor δ for the future, should yield the discounted sum of utilities $\sum_{n=0}^{\infty} \delta^n x = \frac{x}{1-\delta}$.

one side accepts an offer, a simple description of the game structure is summarized by Figure 1.

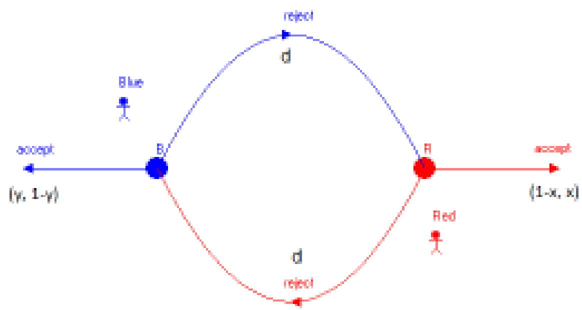

Figure 1: The Simplest Bargaining Game

Since it is always better to get something at some time rather than nothing (ever) there should be an offer, say x by Blue, that is optimally accepted (by Red) at the next turn.[3] In making the offer x Blue rejects (today) an existing offer y by Red.[4] Since Blue would keep $(1-x)$ when Red accepts x (tomorrow) rationality requires for Blue, from today's viewpoint (denoting the discount factor by δ):

$$y \leq \delta(1-x)$$

or $$\delta x + y \leq \delta \tag{1}$$

Moreover, Red could offer a share y which, if accepted by Blue, would leave Red with at least as much as waiting for offer x tomorrow. However, since the game repeats indefinitely and identically to itself, history has no consequences on available options, beliefs or payoffs. We can therefore seek a SPE in stationary strategies, with each side offering constantly the same share.[5] In a stationary SPE, if x and y are optimal offers in the current turns, they should be optimal offers in the next turns as well. But if y is to be offered and accepted optimally at the next turn, then Red can rationally accept x only if:

$$\delta(1-y) \leq x \tag{2}$$

which is equivalent to:

$$\delta^2 + (1-\delta^2)y \leq \delta x + y$$

Together with (1) this means that y must satisfy:

$$\delta^2 + (1-\delta^2)y \leq \delta$$

or $$y \leq \frac{\delta}{1+\delta} \tag{3}$$

Replacing (3) in (2) then yields for x:

$$\delta \leq x + \delta y \leq x + \frac{\delta^2}{1+\delta}$$

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[3]At this point, we do not consider probabilistic acceptance.
[4]We implicitly assume here that x is not an initial offer. That possibility will be addressed below.
[5]This is not saying that a SPE must be stationary. But it is a reasonable lead to explore.

or $\quad x \geq \frac{\delta}{1+\delta}$ (4)

Exchanging the roles of Blue and Red in (4), since both sides should play optimally:

$$y \geq \frac{\delta}{1+\delta}$$

which, together with (3) yields:

$$y = \frac{\delta}{1+\delta} \quad (5)$$

This leaves Red with share $(1-y) = \frac{1}{1+\delta}$ at the next turn. But the true value of that share today is discounted by δ, thus identical to the offer $x = \frac{\delta}{1+\delta}$ s/he receives. The situation is of course symmetrical for Blue. So, neither side can do any better than making the offer $\frac{\delta}{1+\delta}$ in the expectation that the other will accept it and will offer the same. These constant offers and their immediate acceptance provide a SPE of the bargaining game.[6]

As bargaining turns become more frequent, the discount factor δ approaches the value 1 and the Rubinstein solution approaches the traditional 50-50 split that intuitively arises in everyday human affairs (and was also predicted by Nash's axiomatic approach).

6.2 Bargaining and War

Modern social science often makes the same rationalist assumptions as those that underlie game theory. In the realist paradigm, nation states are assumed to make the strategic choices that best achieve their objectives, in particular those enhancing their security. So, there should be a rationalist explanation for war: why should states fight when they could reach the very same expected outcome through peaceful negotiations?

The most basic model that accounts for the alternative between bargaining and war is pictured if Figure 2. Each side, in turn, can choose to accept a current offer made by the other side or to counter with its own offer while continuing to fight. More complex models can be developed that allow bargaining to be peaceful while its failure leads to war. Other questions will also be analyzed in successive sections.

In Figure 2, a prize of size 1 is to be divided between the two sides. Fighting can bring about victory or defeat, leaving the winner with the entire prize and the loser with nothing. This outcome is pictured as a chance move with three possibilities: victory for Blue, victory for Red or stalemate. The picture is merely an enhancement of Figure 1.

Blue may accept Red's offer y_R at node B or fight for another turn while making the offer y_B. Fighting involves the costs $-c_B$ and $-c_R$ for Blue and Red respectively. At a War node, Chance decides with probability v_B to grant Blue a complete victory, or with probability v_R to grant it to Red. A victory yields the entire prize to the victor in perpetuity. With probability $(1-v_B-v_R)$ a stalemate occurs, leading to Red's turn. Discounting δ is applied to future developments. The situation is symmetric for Red at node R.

[6] This reasoning can be generalized to distinct discount factor δ_B and δ_R. In that case $x = \frac{1-\delta_B}{1-\delta_B\delta_R}$. It is also possible to show that this provides the unique SPE of the game.

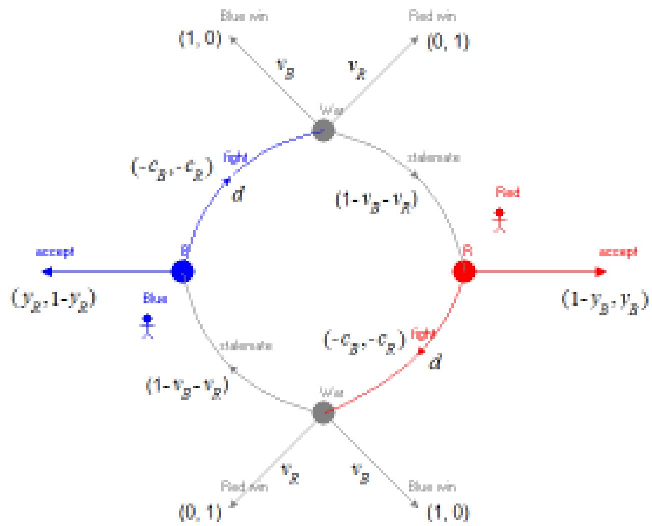

Figure 2: Bargaining and War

Rubinstein's reasoning extends to this situation with some additional definitions and notations. Since rejecting an offer here implies to continue fighting, it involves both immediate payoffs and chance outcomes as well as an expectation of one's opponent's next choice.

The first part can be summarized in the quantity $-c_B + dv_B\frac{1}{1-\delta}$. The second part, should R accept B's offer, would yield $\frac{1}{1-\delta}(1 - y_B)$ discounted by δ and with probability $(1 - v_B - v_R)$. Accepting Red's offer would instead yield $\frac{y_R}{1-\delta}y_R$.

For simplicity of notation, it will be useful to multiplying all quantities through by factor $(1 - \delta)$. We let

$$k_B = -(1-\delta)c_B + \delta v_B \qquad \text{and} \qquad w = 1 - v_B - v_R \tag{6}$$

with $c_B \geq 0$, $v_B \in [0, 1)$, $v_R \in [0, 1)$ and $v_B + v_R < 1$.

The Rubinstein logic of §6.1 then yields the equation (see Theorem in § 6.4):

$$y_R = k_B + \delta w(1 - y_B) \tag{7}$$

and symmetrically for Red.

As illustration, let $v_B = v_R = 0.02$, $\delta = 0.9$, and $(1-\delta)c_B = (1-\delta)c_R = 0.02$. Equation (7) then yields $y_B = y_R \simeq 0.462446$. Entering these data into GamePlan yield the solution displayed in Figure 3.

One should note the critical fact that makes this a MPE of the game: at each decision node, the deciding player finds it indifferent to accept or to continue fighting since the expected payoffs are the same. So, should Red decide to lower the offer y_R, Blue would find it better to continue fighting. As a result, Red's expected payoff of fighting would decline, and accepting y_B would still be best. And should Red increase the offer y_R, Blue would of course accept it with a worse result for Red of continuing to fight. In either case, Red is best off offering y_R and accepting y_B.

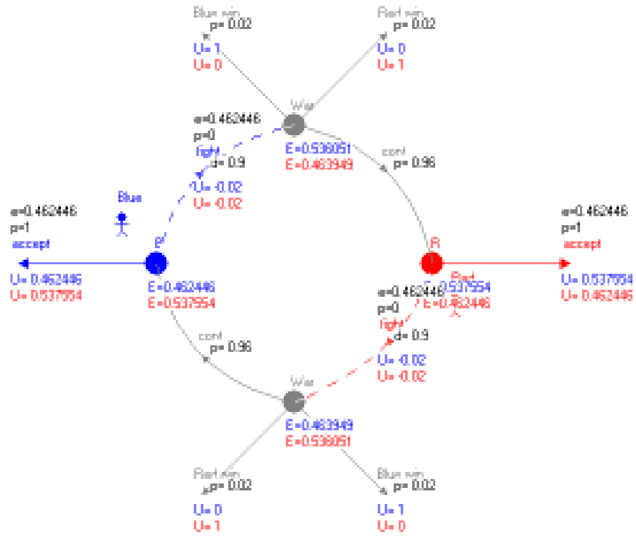

Figure 3: The Bargaining and War Solution

Equation (6) and its various generalizations, that we will encounter in the next few sections, summarizes a simple logic: given that the game structure remains unchanged from one period to the next, the best offer that a player can make should leave him/her *indifferent* between accepting the current opponent's offer or continuing the game, whatever future that involves. What makes this indifference special is the fact that the offers are accepted with certainty. Indifference is normally required for probabilistic play. But here, it is required by two facets of optimality in the offers: (1) a higher offer would be surely accepted leaving the side that makes it with a lower share; and (2) a lower offer would be surely rejected. But the continuation of the struggle is actually worse for the side making the rejected offer. Indeed, looking at the expected payoff $\mathbb{E}U_R(\text{fight@}B)$ for Red of a choice of fight by Blue at node B, one gets:

$$\mathbb{E}U_R(\text{fight@}B) \simeq -0.02 + 0.9 \times 0.463949 \simeq 0.397554 < 0.537554$$

thus providing a much lower expected payoff to Red than sticking to the accepted offer $y_R \simeq 0.462446$ that yields $(1 - y_R) \simeq 0.537554$. This is actually a general fact for a wide class of games involving bargaining as will be discussed in §6.4.

This analysis yields an important conclusion about rationalist explanations for war: if the two sides are completely informed about capabilities and priorities, and if they can make offers at their turn, then there exist optimal offers that will end the dispute right away and will therefore prevent any fighting. So, why do people and nation- states fight when they could negotiate in peace? Several explanations have been proposed that we will discuss in the next few sections.

6.3 The Commitment Problem

Until this point, we have made an implicit assumption: *a deal is a deal*. If this can be true in a society governed by the rule of law, it is far less convincing in the anarchy of international relations. When Nazi Germany re-militarized the Rhineland in 1936 it violated the terms of the Treaty of Versailles that ended WWI, with the ultimate consequences we know. So, how good is a deal if there is no higher authority to enforce it? The models of the previous sections assume that a deal is final since accepting an offer ends the game entirely and forever. But it is not difficult to modify this assumption and to explore its consequences.

As a first step, consider the solved game of Figure 4. The left side is the same as in Figure 1. But the right-hand side is an equivalent formulation of the "final deal" idea.

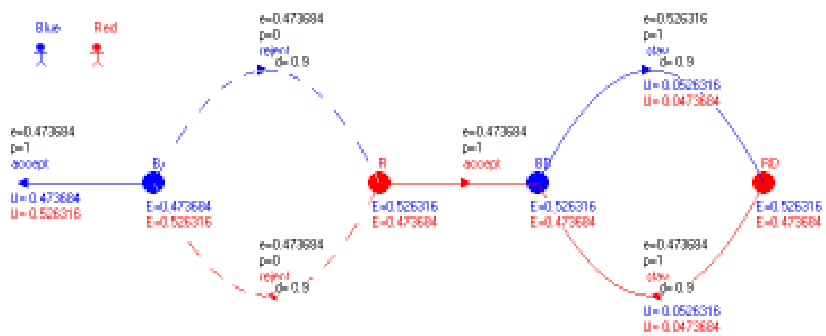

Figure 4: An Equivalent Final Deal Formulation

With $\delta = 0.9$, one finds that the Rubinstein solution is $x = y \simeq 0.473684$. It is what Blue is offered at node B. But instead of being offered such a "final deal" at node R, Red is asked to enter a loop from which there is no way out. In that loop, Red receives a payoff of approximately 0.0473684 each time a move is taken. In a discounted sense, Red will therefore receive $x \simeq 0.473684$, the very same as the final deal value that would appear in the solved game of Figure 1. The interest of this alternative formulation is that it makes more precise what a deal means when it is meant to be enjoyed indefinitely. This also opens the door to generalization.

If a deal is a recurrent enjoyment of a share, then either side in that deal could consider challenging it, evidently for a larger share. Suppose that the right-hand side loop of Figure 4 has been reached and that Red feels it is unfair that Blue enjoys a higher payoff. Would it not be more fair to each enjoy 50% of the prize? After all, this recurring state was reached in a one-time negotiation where Blue implicitly had the advantage of being first to move and offer $x \simeq 0.473684$. Indeed, had Red made that initial offer instead, Blue would have rationally accepted it and Red would now be at a long-term advantage. This is often called the "first mover" advantage in Rubinstein's approach to strategic bargaining. It entirely results from the discounting effect of delays in back and forth bargaining. Of course, it is insignificant if the delay is reduced to almost zero so that the discount factor is almost equal to 1, as happens in many economic interactions. But international negotiations often take months, if not years, and the discounting effect may no longer be negligible. So, how can Red find a peaceful remedy to this perceived injustice? In the game of Figure 5, Red can propose the alternative 50% deal pictured on the right-hand side:

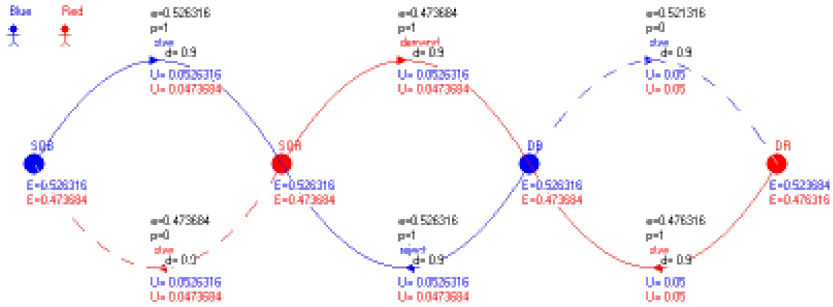

Figure 5: A Peaceful Demand

Since the game is assumed to be in the left loop recurring state, a dissatisfied Red can demand to move to the right-hand side loop where the shares would be equal (the payoffs of 0.05 would accumulate to 0.5 in the discounted stream). Blue can either agree to move to that new status quo or reject the demand. And since the demand is peaceful, the current shares are still enjoyed in the central loop, as long as the right-hand side loop is not reached. There are only two MPEs in this game: either Red never even demands anything and play remains in the left-hand side loop forever; or, as shown in Figure 5, Red makes its demand, but it is always rejected by Blue. In no case will the right-hand-side loop be reached peacefully.

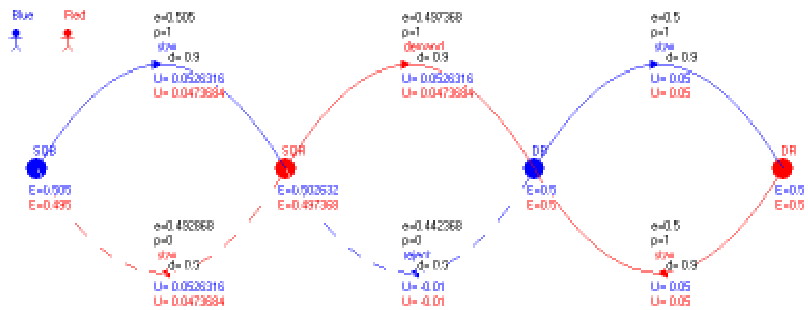

Figure 6: A Not-So-Peaceful Demand

So, any challenge to an existing status quo must come with the credible threat or the actual use of force to have any chance of success. Figure 6 shows how a threat of war can affect the previous outcome. If Blue rejects Red's demand for an adjustment of the status quo, then war automatically breaks out, yielding war costs of -0.01 to each side. As a result, Blue agrees to the new status quo in the unique solution pictured.

But this is a very poor argument. Indeed, there is no bargaining at all in the above picture, and the 50% figure in the right-hand-side loop is completely arbitrary. The real commitment problem is about finding *a deal* that is safe from any credible challenge. In order to address that issue, one must picture the initial bargaining, under the threat of war as in Figure 2, as well as the renewed bargaining under the threat of war that could arise after an initial settlement of the dispute. Figure 7 shows what is perhaps the simplest possible repeated game structure that addresses this issue:

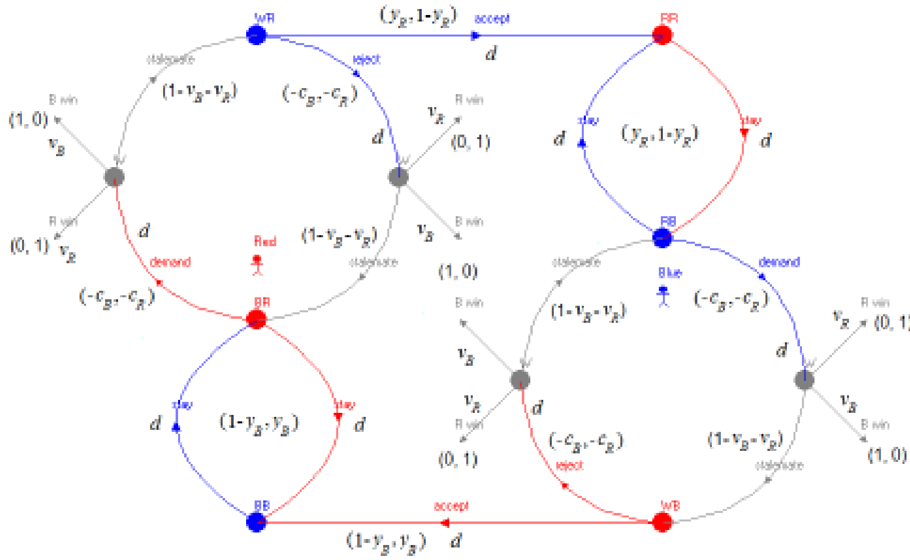

Figure 7: The Commitment Problem

Suppose that some deal $(1 - y_B,\, y_B)$ proposed by Blue has been accepted by Red, possibly after a struggle in the lower-right loop of the game. Assuming that this deal is favorable to Blue, Red might consider challenging it at some future turn, thereby offering y_R to Blue while entering the upper-left war loop of the game. By rejecting that demand Blue would continue the struggle, at least as long as Red perseveres. But if Blue accept the new deal then the upper-right loop becomes the new status quo. In turn, Blue might challenge that status quo with some demand by entering the lower-right war loop. The issue is this: can offers y_B and y_R be formulated such that (1) they would be acceptable as settlement of an existing struggle; and (2) they would discourage any further challenge? Figure 8 shows a typical solution with some arbitrary data.

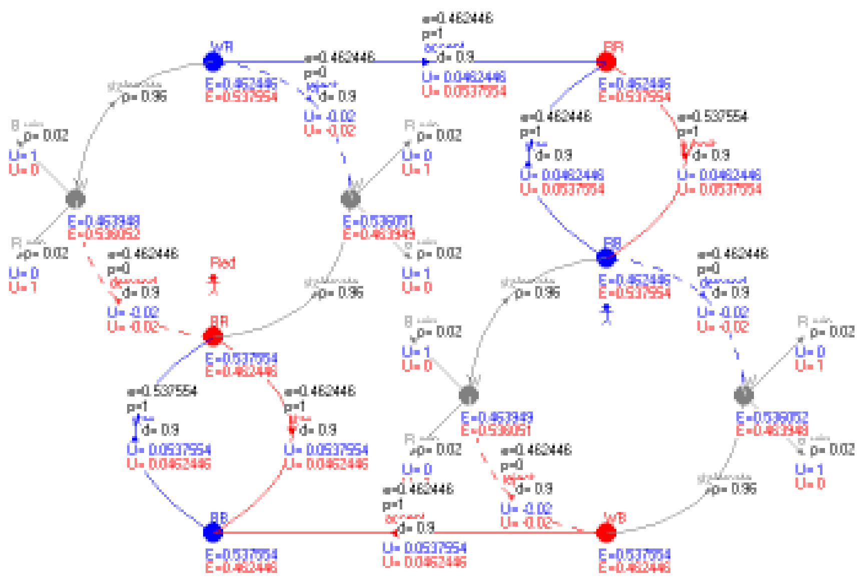

Figure 8: A Solution of the Commitment Problem

The bottom-left peaceful loop is rationally entered by Red from the bottom-right conflict loop since its expected payoff is the same as continuing the struggle with a choice of reject. But after accepting Red has no further interest in making any demand since this would yield an expected payoff that is no better than remaining in the current status quo. The situation is symmetrical for Blue if the upper-right peaceful loop is entered. The particular shares enjoyed in each of the peaceful loops here avoid any future breach of commitments by either side. Also note that a choice of reject could just as well be expected by either side after a peaceful loop is entered. Indeed, Figure 9 shows another solution of the very same game.

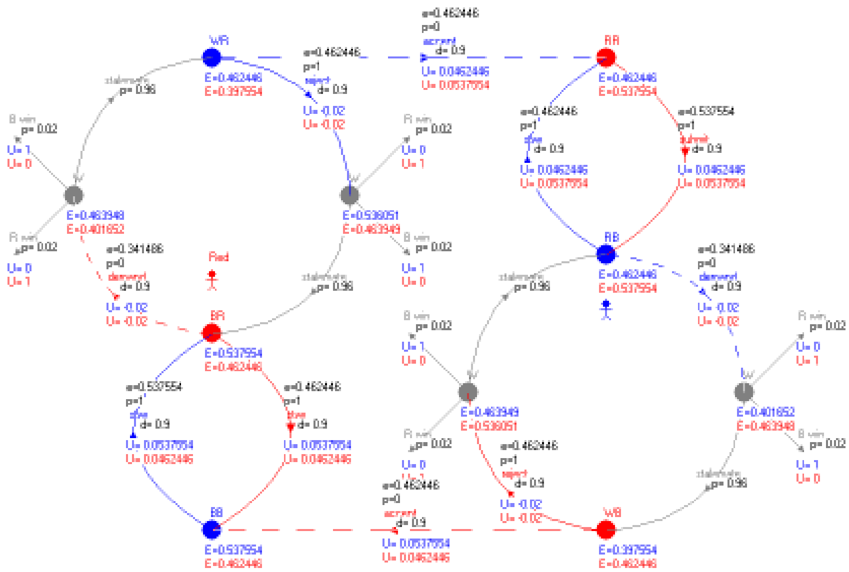

Figure 9: Deterrence in the Commitment Problem

Here, a peaceful loop has been entered as a result of bargaining. But the new expectation of a reject makes it counterproductive to even consider any demand of revision of the status quo. Bargaining can therefore be combined with rational deterrence: as Red accepts the bottom-left status quo in Figure 8, Blue threatens to reject any future demand to reverse that move.

6.4 A General Principle

Many situations can be analyzed in the way of the last section. For instance, one may wonder whether a surprise attack could confer just enough added chances of victory to provide a rational for a military challenge. Or one may consider a peaceful challenge based on a threat of automatic escalation. One may also consider how the developments of an ongoing war could affect the very possibility of bargaining. Some interesting work has been done on battles being fought over forts with victory being achieved only by the side controlling all forts. This issue will be taken up in the next section.

We now lay the groundwork for such generalizations by making some definitions and stating a general result. We assume perfect information throughout. The case of incomplete (and therefore imperfect) information will be taken up in §6.6.

Definition 6.1: A Bargaining and War Game is a two-person stochastic game with discounting. Play alternates between the two sides B and R. Each side can, at its turn, accept a current offer of *status quo* (a division of a prize of size 1) made by the other side at the previous turn. A status quo can take the form of a terminal move or that of a repeating sequence of moves that only the player who accepted it can exit. The game can involve chance moves, including terminal ones, periodic costs or benefits, as well as finitely many states of nature, denoted $S, T \in \mathcal{S}$, that define various costs, benefits and chance moves.

We let y_R^S be the share of status quo offered by R to B at state S. We further let w_{ST} denote the transition probability from state S to state T, $\delta_B \in (0,1)$ B's discount factor, $c_B^S \geq 0$ B's costs, and v_B^S B's probability of victory in state S). We finally let:

$$k_B^S = -(1-\delta_B)c_B^S + \delta_B v_B^S \tag{8}$$

The next definition is inspired from Rubinstein's result and I take the liberty to name a condition after him.

Definition 6.2: A strategy profile in a Bargaining and War game satisfies the *Indifference condition* if the following holds:

(1) Each player always accepts the other side's proposed status quo at its turn; and

(2) Each player is indifferent between accepting or rejecting what is offered at all states S where that player decides, i.e., the following holds at each state S (here for B's turns and similarly for R's turns):

$$y_R^S = k_B^S + \delta_B \sum_{T \in \mathcal{S}} w_{ST}(1 - y_B^T) \tag{9}$$

Theorem 6.1: A strategy profile that satisfies the Indifference condition is a MPE of the Bargaining and War game.

In plain language, if each player is always expected to accept at its turn and is indifferent between accepting or rejecting the proposed status quo, then the offers provide *peaceful optimal* bargains. The proof of the theorem is left as an exercise in §6.7.

What this theorem doesn't say is whether there always exist a MPE satisfying the Indifference conditions in any Bargaining and War game. In fact, the set of equations (9), for all possible states, defines a linear system that does have a formal solution (the set of offers such as y_B). But whether such solutions are non-negative (i.e., are true offers) cannot always be guaranteed. Asimple existence result follows:

Theorem 6.2: Assume a Bargaining and War game such that for all states S, k_B^S is non-negative (where B has the turn at S). Then there exists a strategy profile satisfying the Indifference condition.

The conditions of this theorem are sufficent but not necessary. They are satisfied as long as costs are not too large or the discounting effect is not too high (i.e., the discount factor is not too low). The proof is left as an exercise in §6.7.

Equation (7) is in fact the simplest possible case of equation (9) when war costs and chances of victory are involved. And even (5) is a direct consequence of (9) in the case of peaceful bargaining. We now turn to a typical illustration of (9).

6.5 The Advantage of Position

In this illustration, a war is fought through battles over a set of forts. Only when all the forts have been taken by one side can it claim victory. Battle outcomes are the result of a chance move with known probabilities. In the following example, there are only two forts. If one side holds one fort it only needs to take the second one to win the war. But it may lose that fort to the other side in the next battle. Figure 10 shows a typical stochastic game model and its solution in the form of a MPE satisfying the Indifference condition:

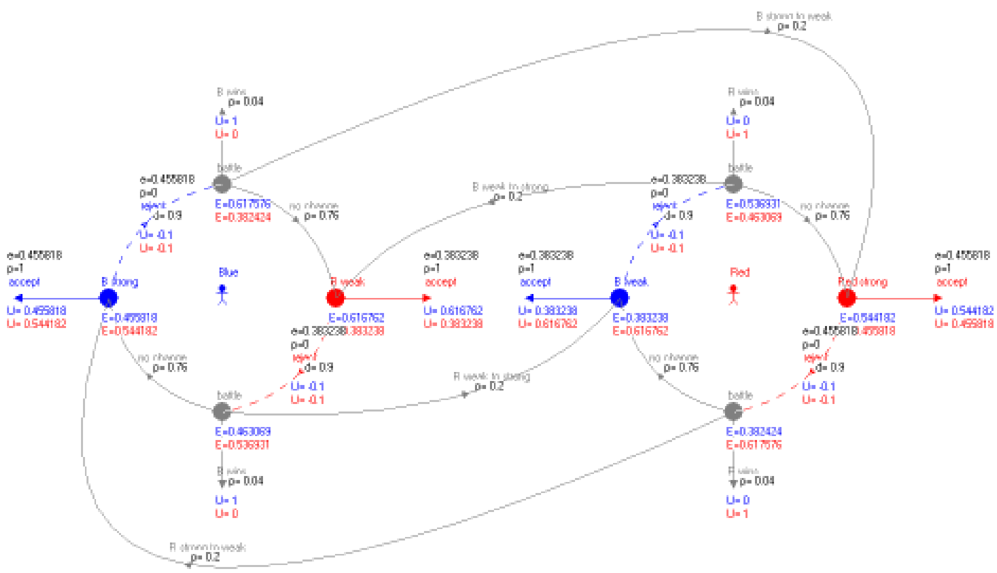

Figure 10: A Battle for Forts and a Peaceful Bargain

In Figure 10, there are two states per player. In the left-side loop B is strong and R is weak, meaning that B holds one fort and is only one fort short of victory. In the right-hand side loop the situation is reversed. At each chance node there are only three possibilities: (1) the strong player wins a second fort and achieves complete victory rewarded by the entire prize; (2) there is a stalemate but the strong player keeps its fort and remains strong; and (3) the strong player loses its fort to the other who then becomes strong. The model could involve one further loop where neither side holds a fort. But much of the essence of the problem is captured in Figure 10.

The question is: does there exist a MPE with the Indifference condition, thereby offering an immediate negotiated outcome? Note that the condition of Theorem 6.2 is not met: $k_B = k_R = -0.1 + 0.9 \times 0.04 = -0.064 < 0$. However, such an MPE still exists in that case (see Exercise 6.7.4). A noteworthy aspect is that offers depend on the current state of the game. B of course receives a higher offer $y_R^S \simeq 0.455818$ when it is strong than $y_R^W \simeq 0.383238$ when it is weak. So, offers are contingent on the current state of the game. Of course, this assumes that contingent offers can be made. Since R implicitly formulates its offer at its turn, it actually needs to formulate two offers: one if B holds the fort and one if R wins it.

6.6 Risk Aversion

Risk aversion is usually formulated as a property of utility functions. A player is risk averse if s/he always prefers the certainty of an outcome to any lottery whose expected result is that same outcome. This preference can be strict or weak. The concept is easily visualized for the interval $[0, 1]$ of our bargaining problem: to any share $x \in (0, 1)$, let our risk averse player assign utility $u(x)$, strictly increasing in x. Consider now a lottery between the outcomes $a < x < b$ with probability p for outcome a (and $(1 - p)$ for outcome b) such that x is the expectation.[7] There is no loss of generality in assuming $u(0) = 0$ and $u(1) = 1$. Figure 11 illustrates the concept.

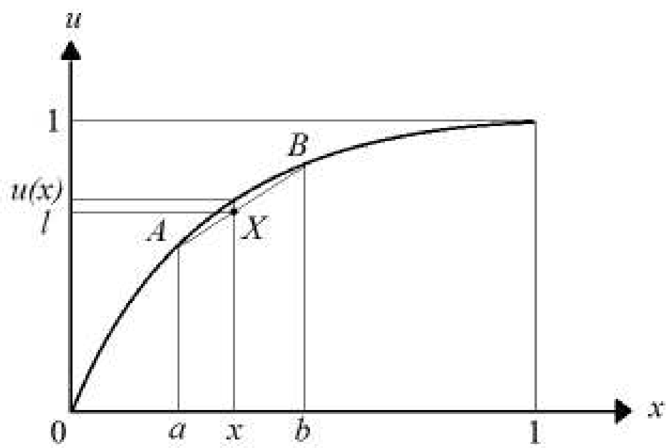

Figure 11: Risk Averse Utility

Point X on the segment joining $A = (a, u(a))$ and $B = (b, u(b))$ represents the lottery with expected result x. The expected utility of that lottery is the vertical coordinate l of X. When $u(x) > l$ for any such lottery the player is (strictly) risk averse. Clearly, this means that the utility curve must lie above any such segment, meaning that it must be strictly concave. Weak risk aversion would instead require weak concavity.[8]

How does bargaining theory extend to the case of risk averse utilities? For simplicity, we assume a symmetric problem with both sides having same utilities and discount factor. The logic of §6.1 can be extended. For an optimal bargain to be reached Blue must be indifferent between accepting y today and receiving $(1 - x)$ discounted tomorrow from Red's optimal acceptance of x:

$$u(y) = \delta u(1 - x) \tag{10}$$

[7]For that lottery to have the expectation x, one needs $p = \frac{x-b}{a-b}$.

[8]A risk-taking player would prefer lotteries to certainties and would have convex utility curves.

And to make the offer y optimally, Red must be indifferent between accepting x today or receiving $(1-y)$ tomorrow from Blue's optimal acceptance of y:

$$u(x) = \delta u(1-y) \tag{11}$$

Since u is continuous and strictly increasing it has a continuous inverse u^{-1} that maps continuously $[0,1]$ into itself. Using this with (10) and (11) one obtains the condition:

$$x = f(x) = u^{-1}\left(\delta u\left(1 - u^{-1}\big(\delta u(1-x)\big)\right)\right) \tag{12}$$

Function f clearly maps continuously $[0,1]$ into itself and must therefore admit a fixed point x (by Brouwer's Theorem) satisfying (12). Then setting $y = u^{-1}\big(\delta u(1-x)\big)$ yields a solution to (10) and (11). It is easy to verify that this solves this generalized bargaining model (also see Exercise 6.8.6).

A typical example of risk aversion is given by utilities $u(x) = x^{\alpha}$ with $\alpha \in (0,1)$. Since $u^{-1}(z) = z^{1/\alpha}$ one gets from (10) and (11):

$$y = \delta^{1/\alpha}(1-x) \qquad \text{and} \qquad x = \delta^{1/\alpha}(1-y)$$

Replacing δ by $\delta^{1/\alpha}$ in (5) one obtains:

$$x = y = \frac{\delta^{1/\alpha}}{1+\delta^{1/\alpha}} \tag{13}$$

Since the function $g(z) = \frac{z}{1+z}$ is strictly increasing in z over $[0,1]$, and since $\delta^{1/\alpha} < \delta$, for δ and $\alpha \in (0,1)$, it follows that risk aversion (with this functional form) *decreases* the magnitude of the optimal offers.

6.7 The Element of Uncertainty

It has been widely argued that fighting rather than negotiating in peace is largely the result of uncertainty. If one side is uncertain about the other side's costs or capabilities, it may be worthwile to make screening offers that will be accepted by the weaker opponent and rejected by the stronger. The question is whether such a strategy is beneficial to the side that implements it.

6.8 Exercises

6.8.1 Distinct Discount Rates

Assume that discount factors have distinct values δ_B for Blue and δ_R for Red. *By retracing the steps* of section 6.1, generalize formula (5) to:

$$x = \frac{\delta_R(1-\delta_B)}{1-\delta_B\delta_R} \qquad \text{and} \qquad y = \frac{\delta_B(1-\delta_R)}{1-\delta_B\delta_R}$$

Hint: replace δ by δ_B in (1) and by δ_R in (2).

6.8.2 Proof of Theorem 6.1

Assume that the Indifference condition holds. Then:

(a) First, argue that accepting the existing offer is always optimal;

(b) Second, argue that increasing one's offer in any state can never improve one's expected payoff (hint: the other player would accept it with certainty);

(c) Third, argue that the sum of the two sides expected payoff from the rejection of an offer is always strictly less than 1;

(d) Fourth, using (c) argue that decreasing one's offer can only decrease one's own expected payoff (hint: the other player would reject it).

6.8.3 Proof of Theorem 6.2

Let $\mathcal{F} : \mathcal{A} = [0,1]^n \to \mathcal{A} = [0,1]^n$ be the n-dimensional map defined as follows: for any vector Y of values y_B^S (or y_R^S) at each state S, let the S^{th} coordinate $f_S(Y)$ of $\mathcal{F}$ be the right-hand side of (9). Using the Brouwer Fixed-Point Theorem, show that $\mathcal{F}$ has a fixed point $Y = \mathcal{F}(Y)$. Why does that provide our result?

6.8.4 Solving the Commitment Game

Write a system of linear equations to reflect the Indifference condition in the game of Figure 7 and solve it to obtain the offers displayed.

6.8.5 Solving the Advantage Game

Write a system of linear equations to reflect the Indifference condition in the game of Figure 10 and solve it to obtain the offers displayed.

6.8.6 Risk Aversion

Generalize (13) to the case of distinct risk averse utilities of coefficients α_B and α_R and discount factors δ_B and δ_R.