Blue, Red and Green
An Arms-Control Dilemma

The three nations of Blue, Red and Green share common borders and their
geography has led to many wars in their common history. But calls by peace activists in
all three states have finally led their governments to negotiate an arms control treaty.

The three states are almost equal in size and military capabilities. The moves by
any one of them, therefore, have symmetrical effects. They also share a common concern
(represented by a discount factor \( d \)) for the future of their relationship.

In the course of their negotiations, they hired GamePlan Incorporated (GPI) as a
consultant to give them some perspective on the possible outcomes of their choices. GPI's
CEO, Professor J.P. Sanctions, provided the following analysis (Coop is short for
Cooperate and means respecting the treaty's terms while Defct is short for Defect and
means reneging on the treaty's terms):

The 3-Way Arms Control Game

Cooperation by all three states would bring desirable stability to the neighborhood
and the enjoyment of a zero (0) utility by all concerned. Defection by all three would
result in the worst outcome measured as a uniform \((-5)\). An individual defector would
enjoy a security advantage of \((+3)\) while the two naive cooperators would suffer a
\((-2)\). A naive cooperator faced with two defectors would suffer an even worse \((-4)\)
while the two defectors would each enjoy \((+1)\). According to Prof. Sanctions, the three
nations would enjoy or suffer such utilities repeatedly, year after year, depending on what
decisions the states made with respect to their treaty commitments.

During the negotiations, various treaty mechanisms have been proposed by the
parties. One multilateral scheme favored by a prominent peace activist can be described
as "weak tit-for-tat" (WTFT): "Always cooperate unless both other sides defect, then
defect once." A lesser known peace activist has been promoting "strong tit-for-tat"
(STFT): "Always cooperate unless at least one side defects, then defect once." A well known political pundit is advocating the "grim trigger" (GT): "Always cooperate until at least one side defects, then defect forever." Prof. Sanctions' private comments on these three proposals are not suitable to be printed here... One of Prof. Sanctions' comments that can be printed concerns the multilateral STFT: "any individual defection would quickly lead to universal defection forever."

You were hired by GPI to work on this project. Prof. Sanctions has asked you to write a report (one that can be printed) on the above three proposals. He also asked you to investigate the feasibility of an alternative scheme inspired from the well known "contrite tit-for-tat" (CTFT): "An individual state will be considered guilty only if it defects on two non-guilty states while they cooperate. All states will be considered non-guilty at first and they will remain so or become so whenever they cooperate. When there is one guilty state, the non-guilty states should defect and will remain non-guilty. Otherwise, a state should always cooperate."

Prof. Sanctions also mentioned an alternative called "probabilistic return to cooperation" (PRC): GTI would be hired (for a fat fee) to monitor compliance. Any defection would lead to a common state of defection (DF) where all sides would be expected to defect. Then, GTI would decide with some fixed probability $p$ whether they all remain in that DF state for one more turn or (with probability $(1 - p)$) whether they "make up" and return to multilateral cooperation (CO). Prof. Sanctions also observed that this scheme can only work with the addition of a clause: if one side fails to defect while in state DF then GTI must let them all remain in state DF with certainty at that turn. The professor is curious (so that he can decide on his fee) about how probability $p$ relates to the discount factor $d$ for this scheme to succeed.

Finally, an ultra-nationalist party figure in the Green state is advocating to pull-out of the talks altogether and to let Blue and Red decide whether to form their own treaty. "They'll chicken out and we will prevail" he is quoted as saying. Prof. Sanctions seems extremely worried about this development and would like you to discuss the issues and possibly make suggestions (he mumbled something like "look at the resulting game between Blue and Red.")

Please write your report on all these issues carefully, as if your job depends on it...

---

1 Note that there can't be more than one guilty state at any one time according to the definition.