

Chapter Two: The GamePlan Software*

2.1 Purpose of the Software

One of the greatest challenges in teaching and doing research in game theory is computational. Although there are powerful theoretical results that ensure the existence of equilibria, few methods exist to calculate them reliably. And when they do exist, their implementation requires computers in all but the simplest games. Moreover, techniques that apply to certain kinds of games are usually not transferable to other kinds. And for many kinds of games there are no wholly reliable methods of obtaining equilibria. But the advent of the personal computer has created opportunities for the design of software that could bring some remedy to this challenge. *GamePlan* was created for that purpose and was designed to meet four basic requirements:

Be User friendly. This means that games must be easily created, edited and solved. This led to a heavy reliance on color-coding for the players, nodes, moves, information sets, and payoffs. It is therefore practically impossible to use *GamePlan* without a color monitor. The user will also greatly benefit from the use of a color printer.

Be Flexible. The data structure and solving algorithms are designed to accommodate almost any game structure that meets the simple requirements that there be finitely many players, moves and nodes. In practice, games cannot exceed some limits due to memory and speed requirements.

Be Exhaustive. There are three solving options (Pure, Explore, and Sample) and three solution concepts (Nash, Perfect Bayesian, and Sequential). The most reliable solving option is Explore, under which the program seeks out all possible solutions of a game, but the most powerful option in terms of speed and exhaustiveness is Sample.

Be Fast. Given the need for exhaustive solving, and despite the optimization of the algorithms, the solving of complex games can be quite slow. It is difficult to give a priori estimates of speed in a rapidly changing computing environment. As the solving proceeds, a dialog box displays the percentage of the game solved so far. This gives constant feedback to the user as to how more time to expect before completion.

2.2 Program Set Up

Gameplan runs under *Microsoft Windows 3.0* and later (including *Windows 95*, *98*, and *NT*). However, the Help file is only accessible in *Windows 3.1* and later. The *GamePlan* version 2.0 program, the *GameHelp* help file, and the library of examples will be automatically set up on your computer hard drive by running the installer program

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gp2setup.exe, which can be downloaded at <http://pws.prserv.net/gameplan>. The installer will also create icons on your desktop and in your program list.

On first running *GamePlan*, you will be asked to enter your name. This will create an eight-character User ID that can be displayed in the Help|Registration dialog box. That dialog box also asks for an Access Code that you may obtain from the above web address by following instructions. If you do not have an access code, the program operates in demo mode. This means that you lack a single function of the program: you cannot save the games you create or edit. And since a game must be saved before it can be solved, you cannot solve those games either. However, you can experience all functions of *GamePlan* separately with the demo: You can create and edit games, and you can solve the library examples since they are already saved. Once you obtain an access code, all functions will be accessible at the same time.

If you open the file *fig101.gp* in the library, for instance, the *GamePlan* desktop will look like Figure 2.01 below.

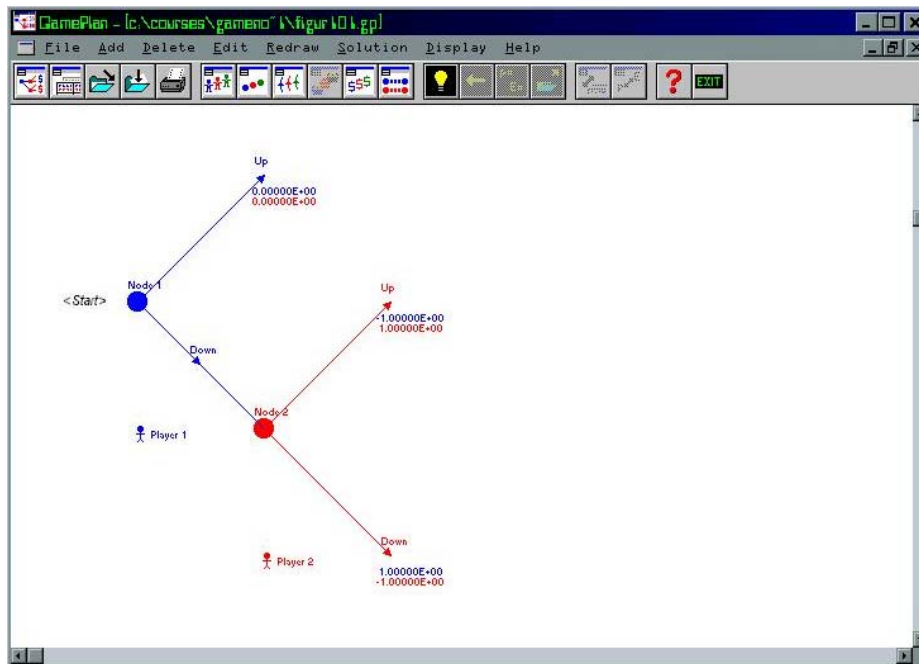








Figure 2.01: The GamePlan Desktop

GamePlan operates just like any other Windows-based program with pull-down menus, speed buttons, and dialog boxes. The File menu together with the first five buttons  provide file operations, printing, and labeling functions. Add, Delete, Edit, and Redraw, together with the next six buttons  and together with the important use of the left button click and double-click, provide all game editing capabilities. The Solution menu together with the next four buttons  provide all solving and solution management and display procedures. The Display menu together with the next two buttons  provide all display capabilities. Finally, the

Help menu provides access to the *GameHelp* help file (as does the  button), the About dialog box, and the Registration dialog box. The exit button  complements the toolbar.

As in many modern programs, menu items and buttons are activated or deactivated depending on the circumstances.

2.3 Game Objects in GamePlan

Game Form. As we saw in Chapter 1, classical game theory distinguishes the extensive form from the *strategic or normal form*, at least as far as "non-cooperative" theory is concerned. The extensive form is the preferred vehicle in *GamePlan*, especially because of the various "perfect" solution concepts. But *GamePlan* also supports the normal form (to a moderate extent, up to three players and a total of twelve strategies), which is the easiest to grasp and the one that has been most widely advertised. In general, the extensive form contains more details, and is therefore better than the normal form. Also, several extensive forms can usually be associated to a same normal form, while the converse is not true (at least up a reordering of strategies).

GamePlan also allows structures more general than the game trees usually considered in classical game theory. In a game tree, there is an initial node from which moves branch out until the final nodes are reached, and payoffs are associated to the final nodes only. *GamePlan* also allows graphs, and payoffs can be associated to final as well as non-final moves. This wider framework is particularly well suited to the representation of many kinds of repeated and stochastic games including those that may not allow any final moves.

As we saw, there are five basic objects that make up the extensive form of a game: players, nodes, moves, outcomes, and information sets. But the normal form can be described by players, (pure) strategies, the "cells" defined by the choice of one strategy per player, and the associated outcomes. The best way to understand the *GamePlan* framework is to open game files (with an extension .gp) from the example library. I will refer to Figure 1.02 (fig102.gp) for a typical extensive form game and to Table 1.02 (tab102.gp) for a typical normal form game, from Chapter 1.

Players. A player is an independent decision maker who faces situations (nodes and information sets) at which he/she can choose between various moves. In *GamePlan*, one may define up to three or four players depending on the game form. Icons suggesting a human shape each followed by the player's name represent them. Each player is assigned a color that will be used to code all other objects (nodes, moves, payoffs, information sets, and strategies) relevant to that player. In the (extensive) game file fig102.gp, for instance, Player 1 is colored in blue and Player 2 is colored in red. Colors are pre-assigned for up to four players (blue, red, green, and orange) in the extensive form, which also allows the predefined Chance (in gray). One may define up to three players in the normal form. The same icons represent them. Each player is also assigned one of the three colors that will be used to code the other objects (strategies and payoffs) relevant to that player. In the game file tab102.gp, for instance, Player 1 is colored in blue and Player 2 is colored in red. There is no predefined Chance in the normal form.

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Nodes. They are the most basic states of an extensive form game and are represented by small colored disks. In classical game theory, there are four types of nodes: decision nodes where a player decides among various possible moves, chance nodes where moves occur with fixed probabilities, independently of the players wishes, final nodes where the game ends, and a single start node which may be a decision or a chance node, but not a final node, where the game begins. *GamePlan* relaxes some of these conventions in order to optimize the representation of certain structures (such as the discounted repeated games). In particular, final nodes are not explicitly represented. Instead, final moves do not lead to any further decision (or chance) node, and they are endowed with payoffs for each player (see below). Nodes are colored according to what player has the turn (or gray when they belong to Chance) and their name appears above the disk in the corresponding color. In Figure 1.02, Node 1 belongs to Player 1 while both Nodes 2 and 3 belong to Player 2. *GamePlan* does not require that a start node (Node 1 is the start node in Figure 1.02) exist, although it will prompt the user to designate one if that is necessary for solving. Version 2.0 allows the definition of up to 48 nodes.

Moves. A move is a transition from one node to another in the extensive form, as permitted by the rules of the game. *GamePlan* represents moves by arcs (lines or curves) endowed with a small arrow indicating the direction of movement that is allowed. The arc starts at the node at which that move is available (to the corresponding player) and therefore bears the same color to indicate which player can exercise it. The name of the move appears in that same color above the arrow. For a non-final move, the arc leads to another node and the arrow indicates the corresponding direction. Non-final moves may be straight or curved depending on how the game was drawn. Final moves can only be straight and do not lead to any further node. In Figure 1.02, the blue moves labeled "Up" and "Down" are non-final while all red moves are final. In the special case of chance moves, which start at chance nodes and are therefore colored in gray, there is an additional requirement: a fixed probability must be assigned to each of the moves. In Figure 1.03, for instance, the two chance moves Up and Down are assigned respective probabilities 0.4 and 0.6 and they appear (in black) above the move name. Probabilities of chance moves must add up to no more than one at any single chance node. Version 2.0 allows the definition of up to 96 moves.

Information Sets. They represent more complex situations that a player can encounter in an extensive form game, when he/she is unsure about some aspects of prior developments or about the exact nature of the opponent(s). Technically, information sets are groups of nodes that belong to a same player. They represent the uncertainty that the player may face at his turn of play: he may not observe with certainty some prior moves that would allow him to decide at which of those nodes he really is. Information sets are subject to some requirements: the moves available at each node must correspond (in name) to those associated to any node in the same information set. This allows the player to make a choice without knowing exactly which move it implies since that depends on which node he really is at. In *GamePlan* version 2.0, information sets are represented by thick dotted lines joining the nodes they involve. In Figure 1.02, the thick dotted line joining Nodes 2 and 3 is an information set. Of course, it has the same color as the nodes it contains.


Strategies and Cells. They appear only in the normal form as illustrated in Table 1.02. Players, strategies, and cells are pre-positioned in the normal form and cannot be moved around as in the extensive form case. The first player always controls the rows by choosing among her strategies whose names appear in blue. The second player always controls the columns by choosing among his strategies whose names appear in red. When there is a third player, she controls the "depth" of the table, as in Table 1.05 (tab105.gp) in Chapter 1, by choosing among her strategies whose names appear in green. There are as many cells as there are sets of one strategy per player.


Outcomes. At the end of each final move in the extensive form, and in each cell in the normal form, there is an outcome of the game that is valued by each of the players according to his/her own preferences. The preference relations are expressed by utilities (also called payoffs), which are numerical values that translate a preference for one outcome over another into a higher value (for the favorite outcome). In *GamePlan*, outcomes are always associated to final moves. But they can also be associated to non-final moves in order to optimize the representation of structures such as discounted repeated games. In Figure 1.02, each of the four final (red) moves shows a pair of numbers underneath their arrow. The numbers are colored according to what player's payoff they represent and are displayed in scientific notation. For instance, the payoff to Player 1 (in blue) at the end of the red move Down that starts at Node 2 is 1.00000E+00 (or more simply 1). In the normal form representation of that game, in Table 1.02, this same payoff is found in the upper-right cell corresponding to the strategies Up by Player 1 and Down by Player 2.


2.4 Creating, Opening, and Saving Games

File Operations. The File Menu contains the usual items:

New creates a blank form to design a game and has two sub menu items:

Extensive creates an extensive form game (including a graph form) and is equivalent to clicking the  button;

Normal creates a normal (strategic) form game and is equivalent to clicking the  button.


Open is equivalent to clicking  and opens a standard File Open dialog box.

Save is equivalent to clicking  and saves the game.

Save As, allows to make copies of a same game under different names.

Print is equivalent to clicking  and opens a standard print dialog box.

Printer Install, opens a standard Windows dialog box to select or install printers.

Exit is equivalent to clicking  and terminates *GamePlan* after doing the usual checkups (saving existing work).

The *GamePlan* File menu also has some specific options:


List shows the technical characteristics of a game in a non-graphic format.

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Title has two sub menu items: Edit opens a dialog box to give the game a name and Redraw lets the user position that title anywhere in the window.

Comment opens a dialog box to create comments that can be positioned anywhere in the game window.


Protection allows the user to seal a file from further editing by third parties by entering a personal password.

Although a game can be created by the choice of New in the File Menu, the beginner may prefer to load an existing game from the example library provided with the program. This is achieved by clicking  and then selecting one of the ".gp" files in the File Open dialog box. Once the file is opened, a game is displayed in the appropriate form (extensive or normal, depending on how it was created) on the desktop. In *GamePlan* version 2.0, only one game at a time can be opened.

As an example, you may open "fig102.gp." The game has two players called "Player 1" (she) and "Player 2" (he). It has a start node called "Node 1" colored in blue because Player 1 has the turn there. At that node, she has two choices: "Up" leads to Node 2 where he will have the turn. That node is therefore colored in red. "Down" similarly leads to Node 3. The two red nodes are joined by a thick broken line (also in red) that denotes an information set. This means that he will not know which of the two nodes he is at when it is his turn to play. Because of this lack of information, he faces the exact same choices "Up" and "Down" at both of his nodes. However, they do not yield the same payoffs since that will depend on Player 1's initial choice. The interpretation of this information set is that his and her choices are simultaneous from the point of view of the information available to them.

2.5 Editing Extensive Form Games

Editing the game "fig102.gp" may be the easiest way to become familiar with the workings of *GamePlan*. In order to not disturb the examples library, use SaveAs from the File menu and enter the name "fig152.gp" at the prompt. Note that the name changes accordingly in the *GamePlan* window. We will use this file as a test bed for editing extensive form games.

Players. There are two distinct but entirely equivalent methods to create, edit, delete, or draw players. One uses the Player items in the Add, Delete, Edit, and Redraw menus. The other method relies on clicking the  (add player) button in the toolbar and double-clicking the player's icon that one wishes to edit, delete, or redraw, thus opening a floating menu. When adding a new player by either method, a dialog box first opens to enter the new player's name. After clicking OK one should press *and hold* the left mouse button and thus drag the new player's icon to its intended position before releasing the mouse button. If the result is not satisfactory, it will be possible to edit or redraw as explained below.

When selecting the Player item in the Edit Menu, one must first select which player to edit from a list box. Once this is done, the Edit Player dialog box opens in which a new name may be entered. Double-clicking a player's icon, and selecting Edit in

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the floating menu, immediately opens that player's dialog box and therefore saves one step.

Delete from either method is similar and opens a confirmation dialog box. However, a player who owns nodes (i.e., has the decision there) cannot be deleted until all such nodes are deleted or have their ownership changed.

Redraw, once selected, waits for the user to press and hold the left mouse button and reposition the player before releasing the button. As an example, you may create a player named "Player 3" that will appear in green. You may then edit its name, redraw its position and finally delete it altogether. But let us keep it in the "fig152.gp" file for later use.

There is a predefined player called "Chance" that represents Nature's moves. Chance has no personal preferences but may probabilistically control moves at Chance nodes. This allows the modeling of so-called incomplete information games. Chance should therefore not be defined by the user and cannot be deleted.

Nodes and Information Sets. Just like for players, there are two equivalent methods for creating, editing, deleting, and drawing nodes. Moreover, both the Edit Menu and the floating menu that opens when double-clicking a node contain the additional Information (Info) item to process information sets.



When creating a new node by selecting Add|Node or by clicking the  button, a New Node dialog box opens as in Figure 2.02 below. One must enter a node name and then select an owner (from the list box) that will be the deciding player at this node. The owner list contains the names of the existing players. You may also check the predefined "Chance" as the node owner. When clicking the OK button, the dialog box closes and the user should press the left mouse button to position the node as desired, and then release it. The node will appear in the color of its owner or in gray if Chance was chosen. The owner is assumed to have the move at this node. If anything is unsatisfactory, it will be possible to edit, redraw, or even delete that node. As an example, you may create a Node 4 in "fig152.gp" and give it the owner "Player 3" that you created above.



Figure 2.02: Add Node Dialog Box

When you close the New node dialog box, a green disk appears in the top left corner of the screen and the mouse pointer changes to a cross which you may position anywhere. When you press and hold the left mouse button, the green disk jumps to where you positioned the mouse. But, before you release the button, you may, holding it down, drag the new node anywhere (the screen will scroll if you move to the edges). When you are satisfied with its position, you can release the button and the pointer will return to its usual arrow, leaving the new node in its fixed place.

Editing, deleting, or redrawing a node is fairly similar to the editing, deleting, and redrawing of players, and double-clicking that node's icon and using the floating menu best achieve it. Deleting also requires that all moves and information sets attached to that node be first deleted or edited so that the node is no longer attached.

Information sets can be created from the Edit|Information menu, or by clicking the  button, or even more efficiently by double clicking a node that already is or will be involved in that information set. In any case, the following Information Set dialog box opens:

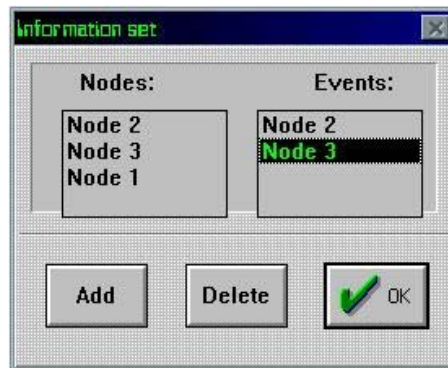



Figure 2.03: Information Set dialog box

The left side list box (Nodes) shows all the nodes of the game while the right side list box (Events) lists those nodes already in the current information set. To add a node to this information set, click it in the nodes list box, and then click the Add button. The node should then appear in the Events list box on the right. However, if the node selected belongs to another player than the current owner of the information set, an error message will appear. Likewise, if the node is already in the current information or belongs to another information set, error messages will be displayed. To delete a node from the information set, simply select it in the Events list box and click the Delete button. A single node in the Events list box cannot be deleted and will not define a true (nontrivial) information set. Once completely done, click the OK button. As long as a nontrivial information set has been defined, a thick dotted line will joint the nodes involved.

For practice, you may double click on Node 4. In the floating menu that appears next to it, click Info. This opens the Information set dialog box with the two list boxes: in the left one are all existing nodes of the game (four) and in the right one, titled "Events," is only Node 4 that you clicked on. Try clicking Node 3 in the left list box and then the

Add button. A warning appears that says, "Node has different owner," since you cannot group such nodes in a same information set. You may then exit both dialog boxes (successively) by clicking OK. To succeed in bringing Node 4 in the information set, you may change the node's owner to Player 2. Do so by double clicking the node again, selecting Edit in the floating menu, and changing the owner. Node 4 then turns red and you can proceed again to bringing it into the info set. However, if you try doing so as before, by double clicking Node 4 and selecting the Info item, you will again get an error message ("Node belongs to another set") when trying to bring in Nodes 2 or 3 into the same information set as Node 4. You will only succeed if you start from Nodes 2 or 3, since they already are in an information set, and then bring in Node 4 into it. However, in order to continue our experiment, let us return to the case where Node 4 belongs to the third player and not to the information set made up of Nodes 2 and 3.

Moves and Outcomes. Just like for players and nodes, there are two equivalent methods for creating, editing, deleting, and drawing moves. However, both the Move Menu and the floating menu that opens when double-clicking the move arrow contain the additional Outcome item to process the players' payoffs.

When creating a new move by selecting Add|Move or by clicking the  button, a New Move dialog box opens as in Figure 2.04 below. Just like with players and nodes, one must enter a valid name. The dialog contains four more objects (a text input line labeled "Discount," a pair of check boxes labeled "final" and "non-final," two list boxes respectively labeled "From" and "Upto," and three buttons "OK," "Payoff," and "Cancel."

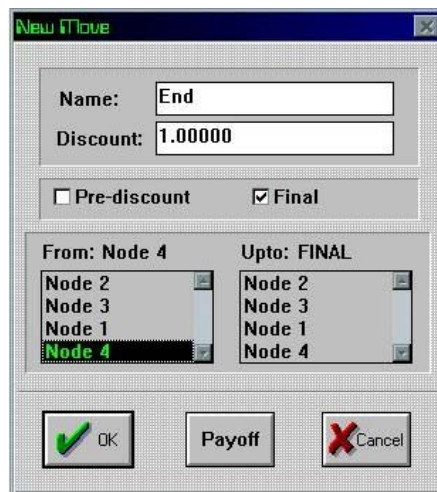


Figure 2.04: Add Move Dialog Box


The From list box allows the selection of the node that will be the origin of the move. Since the dialog box is bulky, the user may wish to move it around as any other dialog box in the program by clicking the box title and dragging the box to a better position. The Upto list box is similar, but a move can be final or non-final, depending on the checkbox selected under the discount line. It is not necessary to check "non-final" before selecting the Upto node of a move since this will be done automatically.

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When "final" is selected, this means that the move leads to a final outcome of the game rather than to another decision or Chance node. From and Upto cannot be the same in *GamePlan*. The New Move dialog box can now be closed by clicking OK. Although one can also click the Payoff button in order to define players' payoffs associated with that move, this is better done later for the beginner.

If the From node is player-owned, the Discount line allows to input a discount factor between zero and one (the default is one). This will be particularly useful when modeling repeated and stochastic games (in later chapters). If the From node is Chance-owned, the same line is headed by the word Proba which is like a discount factor but follows the additional requirement that the sum of all probabilities over the moves from a same chance node must add up to no more than one. Adjusting these probabilities requires a bit of practice and extra care.

Once the move is properly defined and the dialog box is exited by "OK", the program waits for the user to position the move by pressing and holding the left mouse button, then dragging the move to its position before releasing the button. Moves can be curved between From and Upto nodes but can only be straight for final moves. Any unsatisfactory feature can be corrected later with the Edit, Outcome, and Redraw menu items. Editing, deleting, or redrawing a move is similar to the corresponding options for players and nodes. However, deleting a move automatically deletes all payoffs attached to it.

Player's payoffs can be associated to any move (final or not) in *GamePlan*. However, any final move must provide a payoff for each player of the game. To define or edit payoffs, one may use the Edit|Outcome menu, or click the  button, or double-click the move's arrow and select Outcome in the floating menu that opens. The Outcome dialog shows the move's name and the node it originates from. It has a list box for players, text input boxes for payoffs, and three buttons as in Figure 2.05 below.

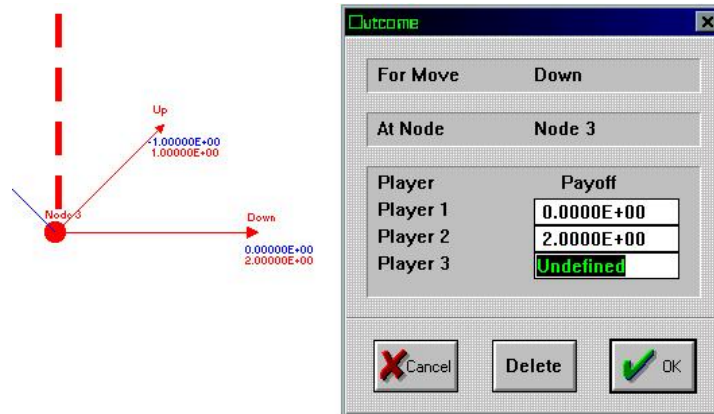



Figure 2.05: The Outcome Dialog Box

To create, edit, or delete a payoff, double-click the payoff box corresponding to a player's name. Then write the new payoff in the payoff box, or press delete to make the desired change. Use the OK button to close the Outcome dialog when done. When the

dialog box closes, the payoffs defined or edited will immediately appear under the name of the move they are attached to and will bear the color of the player they refer to.

2.6 Editing Normal Form Games


Editing normal form games is similar to, but somewhat easier than extensive form games since no drawing is involved. Editing the game "tab102.gp" may be the easiest way to become familiar with the normal form in *GamePlan*. In order to not disturb the examples library, use SaveAs from the File menu and enter the name "tab152.gp" at the prompt.

Players. Just like in the extensive form case, there are two distinct but entirely equivalent methods to create, edit, or delete players. One uses the Player items in the Add, Delete, and Edit menus. The other method relies on clicking the  (add player) button in the toolbar and double-clicking the player's icon that one wishes to edit, or delete, thus opening a floating menu.

When adding a new player by either method, a dialog box first opens to enter the new player's name. After clicking OK the player's icon will appear in a pre-assigned position depending on the order of its creation.

When selecting the Player item in the Edit Menu, one must first select which player to edit from a list box. Once this is done, the Edit Player dialog box opens in which a new name may be entered. Double-clicking a player's icon, and selecting Edit in the floating menu, immediately opens that player's dialog box and therefore saves one step.

Delete from either method is similar and opens a confirmation dialog box. However, a player who owns strategies cannot be deleted until all such strategies are deleted or have their ownership changed.

Strategies. Just like most other objects in *GamePlan*, there are two distinct but entirely equivalent methods to create, edit, or delete strategies. One uses the Strategy item in the Add, Delete, and Edit menus. The other method relies on clicking the  (add strategy) button in the toolbar and double-clicking the strategy name that one wishes to edit or delete, thus opening a floating menu.


When adding a new strategy by either method a dialog box, like Figure 2.06 below, first opens to enter the new strategy's name and selecting the player who owns it. After clicking OK the strategy name will appear in a pre-assigned position depending on the order of its creation. Empty cells will also be created to account for the additional strategy.

When selecting the Strategy item in the Edit Menu, one must first select which strategy to edit from a list box. Once this is done, the Edit Strategy dialog box opens in which a new name may be entered. However, the strategy owner cannot be changed. The only way to achieve this is to delete the strategy and re-create it with a new owner. Double-clicking a strategy's name, and selecting Edit in the floating menu, immediately opens that strategy's dialog box and therefore saves one step.

Delete from either method is similar and opens a confirmation dialog box. Note that all outcomes associated with a deleted strategy are automatically deleted as well, just as with the moves in the extensive form.



Figure 2.06: The Strategy Dialog Box

Cells and Outcomes. Cells cannot be manipulated directly. They appear and disappear according to what strategies are added or deleted. However, the outcomes in cells can be added, deleted, and edited in ways similar to the extensive form case. Player's payoffs must be associated to every cell. To define or edit payoffs, one may use the Edit|Outcome menu, or click the  button, or double-click inside the cell. The Outcome dialog shows the strategies that correspond to that cell. It has a list box for players, text input boxes for payoffs, and three buttons as in Figure 2.05 above.

To create, edit, or delete a payoff, double-click the payoff box corresponding to a player's name. Then write the new payoff in the payoff box, or press delete to make the desired change. Use the OK button to close the Outcome dialog when done. When the dialog box closes, the payoffs defined or edited will immediately appear inside the cell and will bear the color of the player they refer to.

2.7 Display Options

The display menu allows some variations in the way a game and its solutions are displayed. They are especially useful in the extensive form where a lot of information must sometime be shown in very little space.

Names is checked at startup and means that all names of players, nodes, moves, and strategies will be displayed.

Discount is only usable in the extensive form case and allows turning on and off the display of the discount factor associated to each move.

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Player.. allows the selective display of payoffs and expectations associated to the selected player.


All players, the default option, returns to the display of all players' payoff information at the same time.

Comments turn on and off the display of all comments.

Probability turns on and off the moves' probabilities in the display of solutions.

Beliefs, only available in the extensive form, turns on and off the display of beliefs at the node of every information set in the solutions.


Expectation turns on and off the display of expected payoffs associated to moves or strategies in the solution display.

Probabilities and expectations cannot be displayed simultaneously. But one can easily toggle between the two by using the  button that only becomes active when displaying solutions.

Color allows the user to turn off all colors and display the game in black and white. This is especially useful when printing to a non-color printer.


Grid, only available in the extensive form, displays a grid that helps in positioning game objects accurately.

Zoom allows three sizes in which to display the game.

Shift focus, only available in the normal form, allows shifting the depth of the table when there are three players. It is equivalent to clicking the  button.

Scientific allows to display probabilities and beliefs in scientific notation. This can be useful when inspecting sequential equilibria.

2.8 Solving a Game

GamePlan offers three solution methods and three solution concepts that can be selected before solving. The default solution method is Explore and the default solution concept is Perfect. Solving is implemented by selecting the Solve item in the Solution menu or by clicking the  button. The Audit item in that same menu allows the user to check whether there is any fatal flaw in the game (such as some missing payoffs) that would prevent solving. However, that check is automatically implemented when solving.

Solution Methods. By selecting the Method item in the Solution menu, the user can select from three methods: Pure, Explore, and Sample. Of course, the solving will only use the concept of equilibrium selected (Nash, perfect Bayesian, or sequential).

If Pure is selected, *GamePlan* will search exhaustively for all equilibria that only involve "pure" moves or strategies (each has probability zero or one). This method is well suited to some special models such as those inspired by the Theory of Moves. It is also one of the fastest methods and a good preliminary exploration tool.

If Explore is selected, *GamePlan* will search exhaustively for equilibria involving both pure and probabilistic moves. It may overlook some solutions, especially in the case

of a continuum of equilibria or when several moves yield identical payoffs to a player. This is the default method that is checked in the menu at startup.

If Sample is selected, *GamePlan* will search quasi randomly for all equilibria it can find. This is a somewhat less reliable solution method but it usually produces at least one solution quite fast. Also, in the case of a continuum of solutions, Sample may reveal some equilibria that Explore could overlook.

Solution Concepts. The perfect Bayesian ("Perfect") is the default solution concept but one may select "Nash" or "Sequential" instead. Since a sequential equilibrium is also perfect Bayesian, which is also Nash, there are often more solutions under the Nash than the Perfect, and under the perfect than the sequential concepts. For complex games, or when the Nash concept is selected, *GamePlan* arbitrarily cuts off the search after finding one hundred equilibria in order to prevent a system crash due to memory overflow.








Once solving is initiated by clicking  or selecting Solution|Solve, the solve dialog box shown in Figure 2.07 opens. Also note that most buttons and menu items become inactive while others become active. Among the few that remain active are the  and  buttons, while the ,  and  buttons become active. This is the "solve" mode.



Figure 2.07: The Solve Dialog Box

It shows what solving options have been selected and the progress made in solving (% done and number of solutions found). The Abort button stops the solving process although all solutions found so far will still be available. The Solving dialog box is "non-modal" which means that it doesn't capture all the attentions of your computer. You may perform other tasks, such as using a word processor, while *GamePlan* is solving a relatively large game (although this might slow down both activities). In particular, you can display and inspect solutions as they are found.

Showing Solutions. To show solutions, you may either select Solution|Show in the menu or click the  button. This opens the Solution Display dialog box shown in Figure 2.08 below. That box is also automatically displayed when solving is complete.

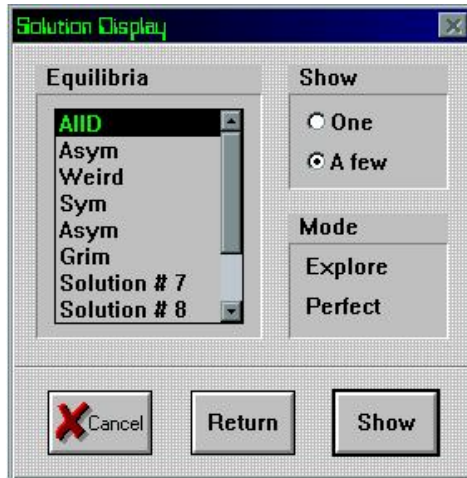


Figure 2.08: The Solution Display Dialog Box

You may display solutions one by one or up to four at a time by checking One or A few in the Show box, selecting the equilibrium to display (alone or first) in the Equilibria list box, and clicking the Show button. *GamePlan* then displays each of the solutions selected in a separate window. Each solution windows may be sized, closed, or minimized. For instance, Figure 2.09 below shows the only solution of the game of Figure 1.05 (fig105.gp) from Chapter 1.

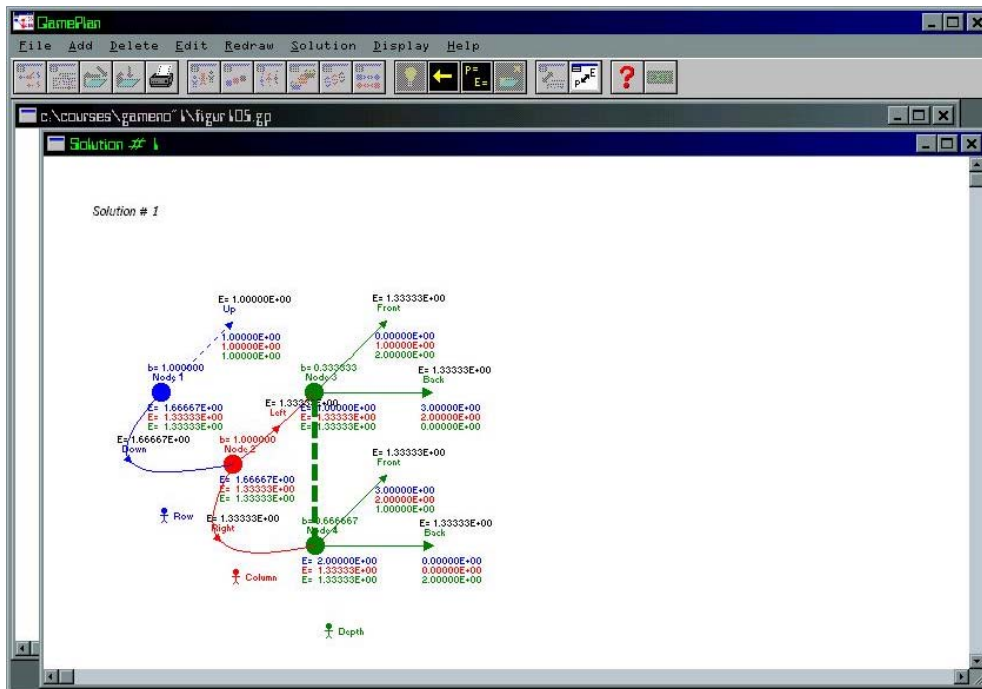


Figure 2.09: The Solution Window

Applicable Game Theory

Reading the Solution Display. The concept of expected payoff is central to the display and to all calculations in *GamePlan*. Recall how it was defined for a game such as that of Figure 2.09. An equilibrium can involve probability distributions over the moves outgoing from any node. For instance, the move Front from Node 3 has probability $2/3=0.66667$ as shown in Figure 1.06. So, the expected payoffs at Node 3 are:

$$E=(2/3)\times 0+(1/3)\times 3=1 \text{ for Row;}$$

$$E=(2/3)\times 1+(1/3)\times 2=1.33333 \text{ for Column;}$$

$$E=(2/3)\times 2+(1/3)\times 0=1.33333 \text{ for Depth.}$$

They are shown just below Node 3. Other expected payoffs are similarly shown under each node in the solution display. They are essential to understand the optimality of the probabilistic or deterministic moves specified in the solution. For instance, Column at Node 2 faces the choices Left and Right which lead to Nodes 3 and 4 respectively. But since Column expects $E=1.33333$ at each of these nodes, either move appears as a best reply given Depth's probabilistic intentions in the solution. Indeed, these expectations from the moves Left and Right are also displayed in Figure 2.09 above the moves' names.

So it is easy to judge whether the solution shown is indeed an equilibrium: a move can have a positive probability (such as Left and Right here) only if it provides the deciding player with the best expectation at the node they originate from. To perform that check, one only needs to toggle between the probability display (as in Figure 1.06) and the expectation display (as in Figure 2.09).


But one must also understand how the move expectations are actually calculated. Indeed, the same kind of check can be performed on Row: since Column uses Left with probability $1/3$ and Right with probability $2/3$ according to Figure 1.06, while Row can expect $E=1$ at Node 3 and $E=2$ at Node 4 by the above calculations, he can expect $E=(1/3)\times 1+(2/3)\times 2=1.66667$ at Node 2. This is also shown just above the move Down in Figure 2.09. Of course, Row receives (with certainty) a payoff of 1 should he choose Up. So, the decision to choose Down with certainty (probability $p=1$) as shown in Figure 1.06 is fully rational in anticipation of Column's and Depth's behavior.


The argument for Depth is a bit more intricate. The expectations displayed atop her moves involve the final payoffs and the probabilities that those moves are actually taken when the corresponding choice is made. Remember that when Depth chooses Front she doesn't know whether she is at Node 3 or 4. But she has beliefs about it. If Column indeed plays according to the probabilities displayed in Figure 1.06, then she should be at Node 3 with probability $1/3$ and at Node 4 with probability $2/3$. So, the choice Front should yield Depth an expectation $E=(1/3)\times 2+(2/3)\times 1=1.33333$ displayed above *both* corresponding moves (Front at Nodes 3 and 4). The expectation for the choice Back is obtained similarly and turns out to be the same. So, Depth can use the given probabilistic choice since it yields her best expected outcome.

Controlling the Solution Display. The main game window and the solution windows can carry a lot of information packed within little space and displaying it all at once could be counterproductive. So, the Display menu allows the user to select what information to display by checking on and off the various items as noted above. Some items, however, cannot be simultaneously checked.

Applicable Game Theory


Quite often, the solution that is displayed has some informations entangled within each other. For instance, the probabilities of moves could be overwritten with the expectations of nodes. If turning on and off the various types of information is not enough to clarify the picture, the user may also redraw the moves in the solution window (this is the only editing type move that is allowed in solve mode). For instance, the moves Down and Right have been redrawn in Figure 2.09 in order to separate well the probabilities of those moves from the expectations at Nodes 2 and 4. To redraw a move, simply double-click its arrow and select Redraw in the floating menu. Then hold the left mouse button and drag the move to a new position before you release the button.

To close the solution display and return to "edit" mode one must click the  button or select Solution|Return in the menu. However, this is not possible while solving since one must either wait until the end of the process or stop it by clicking the Abort button in the Solving dialog box.

Loading and Purging Solutions. *GamePlan* automatically saves all the solutions that it finds in the game file so that they can be inspected during, right after, or long after solving. If solutions exist in the game file, the  button and Load menu item in the Solution menu becomes active. Clicking either will open the Solution Display dialog box. A game that has been solved can no longer be edited unless the user discards the solutions by selecting Purge in the Solution menu.

2.9 Help

The help file is only accessible in Windows 3.1 and later (95, 98). It can be accessed in the group window (if installed) or from inside the program in the Help Menu. It gives some basic information on game theory and *GamePlan* commands.

The Help|Contents item, or a click of the  button opens the help file. *GameHelp* works the same way as any windows help file. It gives summary information about the program features. The Help|About item displays the About box that also appears at the program start. The Help|Registration item opens the Registration dialog box.

Homework

1. Create and solve in *GamePlan* the normal form of the game of Figure 1.03 in Chapter 1. Compare and discuss the solutions.
2. Create and solve in *GamePlan* the extensive forms that you found in exercise #3 of Chapter 1.
3. Create and solve in *GamePlan* the extensive forms that you found in exercise #6 of Chapter 1.
4. Create and solve in *GamePlan* Selten's Horse in exercise #3 of Chapter 1. Also create its normal form and solve it. Comment.